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Abstract. The remarkable degree of accuracy with which some of the Old Kingdom pyramids are oriented towards the cardinal directions is one of the most challenging problems in the history of science. The progressive deviation of the orientation of the 4th Dynasty pyramids from true north was long understood to be a consequence of the pyramids having been aligned to a star whose celestial position changed due to the effect of the general precession of the rotational axis of the Earth. Instead of a single star, recent proposals considered a possible orientation towards some notable vertical or horizontal stellar configurations. The main idea behind these recent attempts at explanation was to justify the gradual deviation of the pyramid alignments by way of the selected target stellar configuration exhibiting a similar azimuthal trend. Considering conventional Egyptian chronologies of this period to be only relative, and the astronomically determined data to be fully reliable, the researchers tried to make the two trends match perfectly by shifting the conventional Old Kingdom chronologies by some, often significant, number of years. Too little attention, however, was paid to allowing for systematic and random errors in the surveying of stars and in the orientation of pyramids towards the observed asterism, which may obfuscate the real accuracy of the methods and conceal the actual targets of observations. In this text, we consider recent proposals and analyze their errors. We propose and discuss two new solutions whose systematic errors are minimal among all the known proposals: one based upon the horizontal alignment of Alioth and Mizar, and another one upon the vertical alignment of Kochab and ζ UMi. In contrast to other methods, the latter pair has the advantage that it could have been observed at lower altitudes. Both variants show an impressive degree of agreement with the trend in the orientation of the pyramids for von Beckerath's (lower estimates) as well as for Baines and Malek's chronologies of the period. It appears to us that the preserved Egyptian astronomical diagrams are fully consistent with our new proposals.

Chronology is the backbone of history
(Edwin Thiele 1944, 137).

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1 Cardinal Directions

This text has the limited objective of discussing a number of recent proposals to update the ancient Egyptian chronology on the basis of astronomical methods that may have been applied to orient some of the Old Kingdom pyramids. Not all of the Egyptian temples were oriented along directions which are nowadays considered as cardinal and the question arises whether our modern concept of cardinality always matches directions which were perceived as sacred by ancient architects. Another problem is that the coincidence—if observed—might not be necessarily conceptual: e.g. many temples with eastern orientations built along the Nile were oriented not according to our modern concept of the east-west direction but perpendicular to the course of the river (Wilkinson 2000, 36). A recent survey and statistical evaluation of orientation of the temples of the Upper Egypt and Lower Nubia often confirmed a topographical orientation but showed that solstitial and stellar alignment were also widely applied in temple construction. [2] The survey also provided statistically significant data relating to (not yet understood) orientations in SSE-directions and this can be a neat example demonstrating that what looks like a non-oriented structure nowadays, might have had a sacral meaning in the past.

In contrast to these results, the existing pyramids of the 4th Dynasty (Snofru's, Khufu's, Djedefre's, Khafre's, and Menkaure's) as well as three pyramids of the 5th Dynasty (Sahure's, Neferirkare's, and Unas')[3] seem to be prime examples of using the *modern* cardinal directions and the exceptional precision of their alignment challenges our understanding of methods used by the ancient architects. Table 1 lists the pyramids which will be considered in the following text; the precision of their orientation towards the cardinal directions (for available measurements, in arcminutes) is given in cols. 3–6 where the deviation dA from the corresponding cardinality (i.e. from true east by the north- and south sides and from true north by the west- and east sides) is counted as positive in the clockwise direction and negative otherwise.[4]

Ruler	Dynasty	dA (north side)	dA (south side)	dA (west side)	dA (east side)
Djoser	3				180'
Snofru-Meidum	4	-35.4' ± 0.4'	-23.6' ± 0.4'	-18.1' ± 0.4'	-20.6' ± 0.4'
Snofru-Bent	4	-7.5' ± 0.2'	-4.2' ± 0.2'	-11.8' ± 0.2'	-17.3' ± 0.2'
Snofru-Red	4				-8.7' ± 0.2'
Khufu	4	-3.6' ± 0.3'	-0.5' ± 0.3'	-3.7' ± 0.3'	-3.4' ± 0.3'
Djedefre	4	-51.7' ±1'	-48.4' ± 1'	-50.8' ±1'	-43.9' ± 1'
Khafre	4	-3.8' ± 0.3'	-5.8 ± 0.3'	-4.2' ± 0.3'	-4.0' ± 0.3'
Menkaure	4	20'	32'	25'	12'
Sahure	5				-23' ± 10'
Neferirkare	5				30' ± 10'
Unas	5			17.4' ± 0.2'	17.1' ± 0.2'

Table 1: Orientation of pyramids: deviation from cardinal directions.

[2] Shaltout and Belmonte 2005, 280. The authors have shown that the orientation to the Nile was "six times more frequent than any other direction."

[3] Due to a poor state of preservation, the data on their orientation are much less accurate than those of the three pyramids in Giza. The only known measurement of Djedefre's pyramid is due to Mathieu (2001, 458).

[4] The data for the orientation of the pyramids can be found in the following sources: Djoser's pyramid in Lauer (1960, 99) and Romer (2007, 279), Snofru's Meidum in Petrie (1892, 6), Snofru's Bent in Dorner (1986, 51), Snofru's Red in Dorner (1998, 30), Khufu's in Nell and Ruggles (2014, Table 1b), Djedefre's in Mathieu (2001, 458), Khafre's in Nell and Ruggles (2014, 322, Table 3b), Menkaure's in Nell and Ruggles (2014, Table 5c, the numbers are mean values for different courses), Sahure's in Arnold (1991), Neferirkare's in Žába (1953, 1), and Unas' pyramid in Dorner (1981). It is well known that different casing and masonry levels may show different orientation (Petrie 1883, 37).

This measured precision of the northern orientation apparently contradicts the common modern consensus that the ancient observations were “more qualitative than quantitative” (Neugebauer 1948, 101). But let us ask a simple question: what did ancient Egyptians understand to be the equivalent of the modern term ‘true north’? Nowadays, we understand the north-south direction as a line connecting the points of intersection of the celestial meridian (the great circle running through the North Celestial Pole and the zenith of an observer) with the observers’ horizon circle. The intersection point which lies closest to the NCP determines the north point at the horizon. Thus, the concept of the rotational pole of the celestial sphere is inevitable for the determination of the northern direction. This concept, however, is not known to have existed in ancient Egypt. How can we be sure that our concept of ‘true north’ really corresponds to a Qibla-direction[5] at that time? The old Kingdom pyramids seem to force us to give a positive answer to this question, but their orientation does not tell us whether the intended target was the invisible rotational center of the celestial sphere or a star fortuitously sighted in that direction; the latter orientation could be mistaken for our modern concept of cardinality.

It is clear that celestial phenomena supplied the ancient surveyors with different ideas of how to determine the seasons and time and how to mark such directions as sunrise, sunset, or noon. The burial chambers of the kings provide us with technical terms used to describe the subjects of observation; the so-called *Pyramid Texts*[6] mention some important stars and constellations[7] and classify a subset of stars as the *Imperishable Stars*—the companions of the king’s eternal afterlife existence. These stars were formerly understood as a group of circumpolar stars, but in recent years the idea that they should be identified with those stars visible every night (not necessarily circumpolar) is becoming popular and appears to be well justified.[8] According to Faulkner (1966, 156) ,“the constant reference to his joining their company carries the implication that he [the king] was one of them”:

> The King has gone to the great island in the midst of the Field of Offerings on which the swallow-gods alight; the swallows are the C. S. [Circumpolar Stars/ Imperishable Stars].[9]

> You shall set the King as a magistrate among the spirits, the C. S. in the north of the sky, who rule over offerings and protect oblations.[10]

Atum asks the king:

> There is no star-god who has no companion; have you your companion? Look at me! You have seen the shapes of the children of their fathers, who know their speech, the C. S.[11]

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[5] The terminology was discussed in the classical study of Knut Tallqvist (1928, 117—129, 280). Many important studies on sacral directions can be mentioned here, above all, a huge collection of information in Podosinov (1971).

[6] Such texts are found in the pyramids of Unas, Teti, Pepi I, Merenre, Pepi II, and Neith and in those of the kings’ wives. It is commonly agreed that the Pyramid Texts constitute a corpus that had been in use for some time before it was inscribed in Unas’ pyramid. The texts contain three major groups of spells: the Offering and Insignia Rituals, the Resurrection Ritual, and the Morning Ritual. The first always appears at the north wall of the burial chamber, the second occupies the south wall, and the Morning Ritual (in the four pyramids in which it occurs) is associated with the east. As J. P Allen (2005, 2) emphasized, “Though many of the Pyramid Texts are repeated in each pyramid, research has also shown that each corpus was conceived as a unit, with the texts meant to be read from wall to wall in a specific order”.

[7] Such as *Sah* (corresponds, possibly to a part of the constellation of Orion), *Sepedet* (Sirius), *Meskhetiu* (the Plough or the Big Dipper in the constellation of the Big Bear), *Two Enneads, Two Adzes,* and the *Mooring Post.*

[8] Belmonte (2001, note 18) gives a list of the supporters of this idea.

[9] Faulkner 1966, 156; PT519, 1216.

[10] Faulkner 1966, 157; PT 519, 1220.

[11] Faulkner (1966, 156, §141) comments on this: “Apparently Atum is asking the King if he has chosen his necessary companion (or sponsor?) from among the Circumpolar Stars.”

We have no evidence to clarify whether the perception of circumpolation[12] had led to the epistemological concept of a rotational *center* of the sky,[13] that is, to our modern concept of the North Celestial Pole; some authors have argued that Thuban (α Draconis, apparent magnitude 3.67) was used to define the north direction,[14] but such an assumption attributes to this relatively faint star a very important role not confirmed by textual evidence and even the identification of this star in the Egyptian texts is questionable. More than that, in the range of the widely agreed upon chronologies for Khufu's reign (2589 BC for Khufu's accession date according to Malek's chronology (Shaw 2000, 482) and 2554 BC for von Beckerath's (1997) lower estimate, the maximal azimuthal deviation of Thuban from true north varied between $\pm$ 1° and $\pm$ 1° 40´. With the accuracy of the orientation of Khufu's pyramid of about 4´, one has to assume that not Thuban but the middle point of its circular trajectory, that is, the Pole, was used for the alignment of Khufu's pyramid. But if so, why Thuban? If the concept of the Pole as a rotational center of the sky already existed, it would be easier to sight a brighter star to estimate the center of its circular daily trajectory.

Obviously, if the orientation *intentionally* matched the north-south or east-west direction, the method used for the orientation of pyramids must have been an *astronomical method*, because the concept of such cardinality *per se* is an astronomical concept. With one of the cardinal directions being found, another can be determined as a perpendicular to the former—and it is not always easy to surmise which of two cardinal directions was primarily determined from observations.

Possible explanations for astronomical orientation of the ancient monuments are based upon either solar or stellar observations. The solar observations permit determination of the east/west point on the horizon by bisecting the angle between the directions of the extreme points of sunrise (or sunset) which correspond to summer and winter solstices.[15] Alternatively, the east-west direction can be determined by observation of a diurnal gnomon-shadow using a variant of the method later known as the *Indian circle*: a line drawn through two points on the shadow curve lying at the same distance from the base of the gnomon marks the east-west direction.[16]

The north-south cardinal direction can be found from both solar and stellar observations. The shortest shadow of a vertical gnomon observed during daytime marks the local noon as well as the direction of the local meridian.[17] This direction is also given by the projection of the highest or lowest positions of a star (culmination) onto the horizon of an observer.[18] But the trace of the shadow or the path of a star near the meridian are shallow curves and this makes it difficult to fix the exact position of the extremum (or, equivalently, the exact moment of the

[12] Of course, only a part of the circular trajectory of a star is visible during one night in Egypt.

[13] The Egyptians understood the world as a space bounded by land and sky within a cosmic ocean; correspondingly, the sky was seen as the surface of the ocean where it met the atmosphere. Possibly, the sky was contemplated as an iron container of water, pieces of which fell to the Earth as meteors (Almansa-Villatoro 2020). Thus, the rotational center of the sky might have been surmised as a sort of a central whirlpool (or as the rotational centre of the iron container of the cosmic ocean).

[14] Sir John Herschel (1887) in his *Outlines of Astronomy* (first published in 1847) was the first to attempt to date pyramids astronomically by assuming that their north facing entrance passages indicated the direction of the pole star at the time of their construction and proposed Thuban as a possible target to date the Great Pyramid (the famous astronomer was unaware of the simpler explanation that the tangent of the angle of the slope of the entrance was exactly ½); he was followed by Charles Piazzi Smyth (1874). This idea was also supported by Proctor (1880). Recently, with an inventive interpretation, by A. Puchkov (2019).

[15] The method is discussed by Gallo (1998, 77–90); Belmonte (1999, 275) rejected this method because, in his opinion, it could not provide the required precision. The crucial points are the angular size of the Sun's disc and the effect of atmospheric refraction.

[16] Discussed and tested by G. Dash (2013, 2017).

[17] The method was proposed by E. Zinner (1931, 1–32). It was tested by M. Isler (1989, 191–206) who achieved a precision of 19 minutes of arc using a sort of a *bay* (a sight-seeing device made from a specially cut palm-rib with a sliced V-shape at one end) 60 cm high.

[18] Discussed by Romieu (1902).

event). By observations of the gnomon shadow, another variant of the Indian circle can be used to improve the precision: first, two points on the diurnal shadow curve lying at the same distance from the base of the gnomon should be marked, secondly, the arc between these points should be halved, and thirdly, the point obtained must be connected with the base of the gnomon. By stellar observations, a better precision can be achieved by bisecting an angle between the positions of a suitable star at the same altitude (observed e.g. relative to an artificial horizon).[19]

The understanding that two cardinal directions complement each other must have emerged at a later stage of reflection: to know that the direction towards the highest position of the Sun during its *diurnal* apparent motion (or the highest/lowest position of a star during one night) is perpendicular to the line given by the direction of the sunrise/sunset at equinox, i.e. defined by the Sun's *annual* motion, is not trivial; rather, we assume that historically (together with the north-south axis), the directions towards the points of sunrise and sunset at summer or winter solstices (with the latter marked in ancient Egypt by the heliacal rising of Sirius), were used as the cardinal directions.

In 1984 an important observation was published by S. Haack that the orientation of the pyramids of the Fourth Dynasty follows a special pattern of digression from true north. This is displayed in Fig. 1 where the y-axis gives the azimuths[20] of the east sides of the pyramids and the time-axis follows von Beckerath's chronology;[21] the trend, of course, exists also for other chronologies although the gradients of the lines become slightly different.[22] Following K. Spence (2000, 320), it is assumed here and further in the text that "the pyramid alignment ceremony occurred in year 2 of each king's reign (with the exception of those for the later pyramids of Snofru), after the burial of his predecessor, the choice of a suitable location, and preparation and levelling of the site."

[19] Proposed by Edwards (1947, 209–211).

[20] Azimuth is an angle counted from true north clockwise from 0° to 360°. In this text, following the historical tradition, the azimuths to the west from true north will be given as negative [0, -180°] and to the east as positive [0, 180°] numbers. With this counting, azimuth becomes equivalent to the deviation of a direction from true north. In this graph and further in the text we use the mean values of the azimuths of the pyramids' sides given in Table 1 with the confidence intervals kept in mind.

[21] We use here the lower range of dates given by von Beckerath (1997). The length of Snofru's reign of 48-years follows Stadelmann (1986; 1990); the same duration for Snofru's reign is accepted by Schneider (2002, 278). The start dates for construction of the pyramid at Meidum, the Bent and Red pyramids follow the temporal proportions given in Stadelmann (1986) which were also used by Spence (2000).

[22] Throughout the text, we exclude the data for Khafre's pyramid in the calculation of the trendlines; the grounds will be explained in the following section.

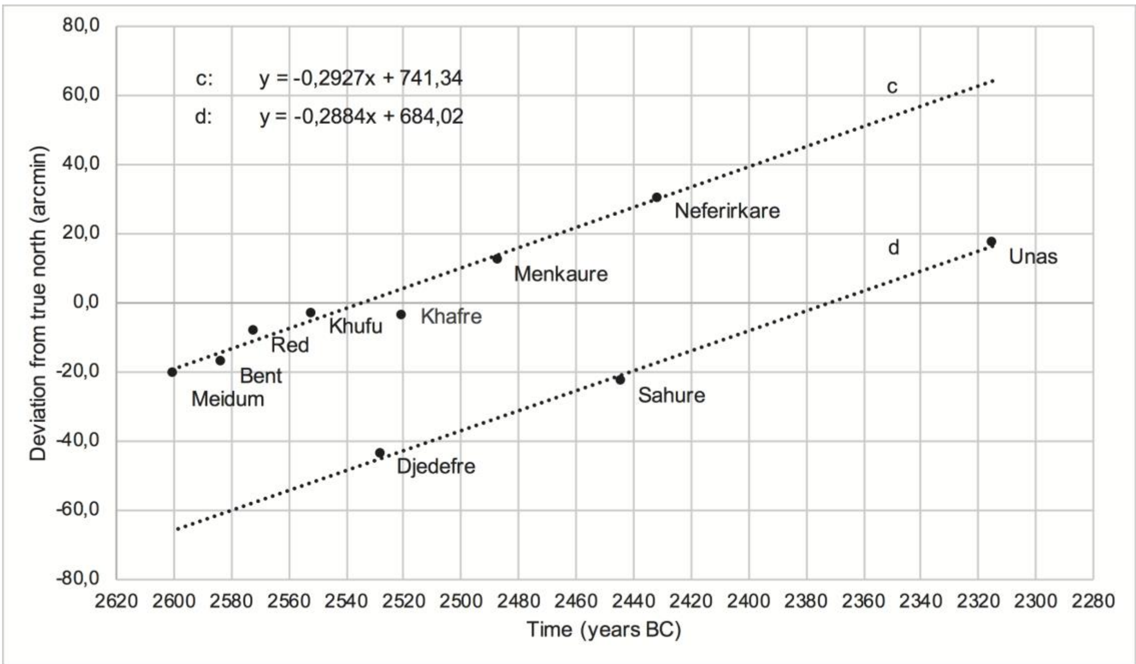

Fig. 1: Orientation of pyramids: deviation from true north over time.

What S. Haack identified as a temporal correlation of the orientation of the pyramids was the trendline ‘c’ (which we will call the ‘main’ trendline) running as a guide to the eye through the azimuths accurately surveyed at that time Snofru’s, Khufu’s, Khafre’s, and Menkaure’s pyramids; the azimuths of Djedefre’s, Sahure’s, and Unas’ pyramids lie along the line ‘d’ as shown in Fig. 1 (the ‘secondary’ trendline).[23] From the moment of publication onwards, a considerable effort was made to explain this apparent trend in the alignment of the pyramids. In fact, all the proposals relied heavily on an unspoken assumption which we will formulate as follows:

Assumption 1: *The pattern of deviation from true north over time identified for some pyramids is not accidental, but caused by the alignment procedure itself.*

Assumption 1—although not justified mathematically—is a very useful guideline because it permits refutation of the astronomical methods which do not show a secular trend over time.[24] We can exclude, for example, the Celestial Pole as a possible target of observation—the position of the Pole is fixed for a given position on Earth and changes only due to geophysical factors which are irrelevant here.[25] We can also exclude solar observations: methods based upon observing the direction of the Sun at the moments of its equal elevation above the horizon (rising and setting are special cases of this configuration) followed by halving of the observed arcs or angles, do not show a secular trend. The Sun could also not have been observed at culmination, at its highest position, because at that moment the Sun crosses the meridian of the location, that is, always marks the true north-south direction.

[23] S. Grigoriev (2015, 2) was possibly the first to point towards the existence of the trendline in the orientation of Djedefre’s, Sahure’s, and Unas’ pyramids.

[24] An increasing accuracy of alignment methods and observations would also manifest itself as a secular trend. We should, however, reject this explanation on the ground that the accuracy of the apparent northern orientation was not increasing with time but reached its peak at Khufu’s and Khafre’s time decreasing afterwards.

[25] For example, the continental drift with the actual figure for the African plate (ca. 2.15 cm per year in a general north-east direction) and the rotation of this continent (0.3°/My) cannot change the northern alignment of pyramids at the surveyed rate.

Thus, we can assume that astronomical orientation of pyramids was performed using stellar observations. Excluding observations which do not show a secular trend over time, we can argue that (exactly as in the case of the Sun) the alignment procedure could not have been based upon the observation of stars at equal elevations or at their upper or lower culminations.

Within a certain range of accuracy, two effects (besides the daily rotation around the North Celestial Pole) change the apparent stellar positions—the proper motion of stars, and the general precession of the rotational axis of the Earth. As the Celestial Pole shifts, there is a corresponding gradual shift in the apparent orientation of the whole star field with respect to the NCP; without proper motion of stars and the precession of the ecliptic, after approximately 26,000 years the NCP would occupy the same position relative to the stars, as viewed from a particular position on Earth. The apparent effect of precession for an observer at a given geographical location depends upon the right ascension and declination of stars; it manifests itself for every particular star and for every stellar configuration differently.

Assuming that over the time of the Old Kingdom, general precession was responsible for the secular trend in the alignment of the pyramids, S. Haack proposed that the primary alignment direction was true east, which was determined by observation of β Scorpii as first visible at its rising. Such an explanation presumes, however, that this cardinal direction was already known: otherwise, one cannot explain why the orientation of the pyramids was based upon adjustment to a relatively faint, not very prominent star in the east.[26]

Rejecting on those grounds the possibility that true east was found by observing a certain star at its rising (there was also no especially prominent or bright star rising close enough to this direction), we can conclude that neither the northern nor the southern sides of the pyramids were oriented along the east-west direction but rather the eastern or western sides, or both of them, were oriented with the help of stellar observations.

We should now ask ourselves: if not at the time of rising and setting, not at the same elevation, and not at culmination, where or when should a chosen star have been observed? In any case, a star could not have been observed at an accidental position or at an arbitrary moment of time (in this case, the alignment would not show any trend); the positions should have been specified by some rule. This unavoidably leads us to conclude that *the stars used for alignment of the pyramids must have been observed at a particular stellar configuration.*

So far, we have considered the consequences of the Assumption 1 for the astronomical orientation of pyramids. Less obvious in studies concerning this subject was the following hidden assumption:

Assumption 2: *Because the deviations of the pyramid orientation from true north over time all lie close to the same line, the precision of the alignment procedure was very high.*

Provided that Assumption 1 is true, the second assumption seems to be only logical, but it is in no way equivalent to the assumption that the *accuracy* of the alignment was comparably high. Let us remember the difference between these two concepts: whereas *accuracy* shows how close is a particular measurement to the true value, *precision* represents the consistency of measurements and indicates the limit of the method and measuring instrument. For example, let us consider a measured series of the azimuths of the true north direction [350°10', 350°8', 0°10', 0°8']. In this case, we can conclude that the measurements—with the random errors of each measurement varied in the range of ± 10' and a systematic error of 10°—were more precise than accurate. In the case of the pyramids, we can make the following 'Gedankenexperiment.' Let us assume that a target star was observed with an azimuth of 359° 40' (or – 20'), but a systematic error of the observation attained a value of +20´. In this case, the

[26] In addition, Haack (1984, 124) assumes that altitude (in degrees) of a star when it first becomes visible is about equal to the magnitude of that star and that makes the direction (azimuth) of the alignment strongly dependent upon the magnitude of the selected star, that is, fortuitous.

side of the pyramid would be perfectly oriented towards true north and we would speculate about a star which could have been observed exactly in the north direction at the time in question. In this example, the systematic error (observational and/or instrumental) has masked the displacement of the target star from true north.[27]

In principle, the existence of two trendlines with similar gradients in the azimuths of the pyramids (this similarity was discussed first possibly in Puchkov 2019) can be explained as a result of orientation towards the same stellar configuration with a systematic error of about 25–30´. It is the high *precision* of their orientation to true north which tempts us to assume also the high *accuracy* of their orientation, to look for alternative possible explanations and to formulate the following assumption

Assumption 3: *The accuracy of the orientation of pyramids towards the target of observations was very high.*

All the methods aimed at explaining the orientations of pyramids advanced so far are based upon this silent assumption. However, the high apparent accuracy of the northern alignment of the pyramids cannot be presumed *a priori* but should be validated by comparison of the azimuths of their east- or west sides with the azimuths of the true targets used for their orientation.

Due to the effects of precession, stellar azimuths are time-dependent and can only be calculated for specified calendar dates. Several chronologies of the period are available; in this text we have used the four most agreed-upon chronologies—von Beckerath's (1997), Baines and Malek's (1980), Malek's (as given in Shaw 2000) and Hornung et al. (2006), all modified according to Stadelmann's proposal by having 48-years for the duration of Snofru's reign.[28] The corresponding data in years BC are given in Table 2 where the dates in square brackets for the construction of Snofru's Bent and Red pyramids follow the temporal proportions given in Stadelmann (1986).

Ruler/pyramid	Dynasty	Acession date (Shaw)	Acession date (Baines&Malek)	Acession date (von Beckerath)	Acession date (Hornung et al)
Snofru-Meidum	4	2637	2599	2602	2558
Snofru-Bent	4	[2620]	[2580]	[2583]	[2541]
Snofru-Red	4	[2609]	[2569]	[2572]	[2530]
Khufu	4	2589	2551	2554	2509
Djedefre	4	2566	2528	2530	2482
Khafre	4	2558	2520	2522	2472
Menkaure	4	2532	2490	2489	2447
Sahure	5	2487	2458	2446	2428
Neferirkare	5	2475	2446	2433	2415
Unas	5	2375	2356	2317	2321

Table 2: Accession dates according to Shaw's, Baines' and Malek's, von Beckerath's, and Hornung's chronologies with 48 years for the reign of Snofru.

[27] Ptolemy in his *Almagest* (VI 9, H527) mentioned a case of mutual compensation of two errors and called it "a chance stroke of good luck."

[28] In the following text they will be addressed simply as "von Beckerath's," "Baines and Malek's," "Shaw's," and "Hornung's" chronologies. For the pyramids along the main trendline, the lower estimates of Baines and Malek (1980, 36) are very close to von Beckerath's dates and produce practically the same results. The influence of the presumed duration of Snofru's reign on our numerical calculations will be discussed later in this text.

No documents have yet been found that can help us to understand exactly how the architects of the Old Kingdom oriented any of their constructions. Some later texts of the Helleno–Roman period report that foundations of the temples were established during a ceremony called the *Stretching of the Cord* when a primary direction was laid out by stretching a rope between two stakes or poles.[29] For example, the text on the walls of the Temple of Horus at Edfu informs us that the pharaoh and the goddess Seshat were responsible for this technical procedure:

> I take the pole and grasp the handle of the mallet; I grasp the cord with Seshat. I turned my eyes according to the movement of the stars and I let my eyes enter into Meskhetiu. The time-indicator God was standing next to his merkhet. I established the four corners of your temple.[30]

This description is dated back to 237 BC; it follows, however, a very ancient tradition which is depicted, e.g. in the chapel of Queen Hatshepsut at Karnak or in the Sun Temple of Abu Gurab: "the Stretching of the Cords inscriptions surely reflect the very ancient tradition that can be followed across the New Kingdom to the beginning of the Old Kingdom, the moment when we find a relief in the sun temple of Abu Gurab, between Giza and Saqqara where the Pharaoh Niuserre, sixth king of the Fifth Dynasty (c. 2425 B.C.), is seen together with Seshat almost in the standardized image of later epochs, or when later traditions (the so called *Book of the Foundations*) attributed the "invention" of the ceremony to the sage Imhotep, architect, and perhaps "astronomer", of the Third Dynasty King Djeser" (Belmonte (2001, S7). The earliest mention of this ceremony refers to an unknown king of the 1th Dynasty and is presented in the basalt slab known as the Palermo Stone (which records two similar events)—thus, the ceremony may be even older than the pyramids.

An important inference can be drawn from this textual evidence: firstly, that he *merkhet*[31] was somehow used in the procedure; secondly, it was assumed to be used by the goddess of *time* and thus had something to do with marking a specific moment of observation, and thirdly, the asterism *Meskhetiu* played an important role in the alignment. Last but not least: positions of stars change steadily and one surveyor can hardly orient two sides (east and west) of a pyramid by observation of the *same* stellar configuration in the course of one night. The text, however, tells us about marking all four corners of the construction in course of a single ceremony; thus, we can assume that either only one side was oriented astronomically and the others have been aligned to it by geometrical means, or two sides were oriented simultaneously by two surveyors, or that two different configurations of stars were used to align the west- and the east sides of the pyramids independently.[32] Whereas the direction determined

[29] See Belmonte 2001: S. 6–7. Clearly, the dates of the Stretching of the Cord ceremony could not have been fixed according to a solar calendar—the lighting conditions would have been considered and the optimal observation times were at the beginning of every moon month which began on the morning when the waning crescent moon could no longer be seen (Parker 1956, 26). An open question is also what happened if weather conditions had not allowed observations at an appointed day? Could the ceremony be moved to the next possible night, or did one have to postpone the ceremony by one year?

[30] Žaba (1953, 58), the excerpt P1.II A a, translated from French by F. Monnier (from Puchkov 2019). For the very similar description in the Temple of Hathor at Dendera see Žaba (1953, 59, the excerpt P1.II C d). The first source is Brugsch (1891). See also Faulkner (1966, 153—61); Krupp (1983, 211–13); Leitz (1991, 49).

[31] The *merkhet* was made as a bar with a plumb line attached to a wooden or bone handle. While the use of the instrument as a sundial is confirmed by inscriptions on the handle, its applicability in the Stretching of the Cord ceremony is not clear. Two of the instruments in the collection of the Egyptian Museum in Berlin are inscribed with hieroglyphic texts. The text on instrument Nr. 14805 (with a length of 11.5 cm length and width of 1.3 cm) has been interpreted by L. Borchardt (1899, 11) as "knowing the way of the Sun/Moon and the stars," which points towards ,a possible additional application at night time. Later, George (1974, 103) proposed that the text should be read as "two Sun's discs" and placed the second disc into the Egyptian underworld sky. About the accuracy of measurements with a *merkhet* see Tupikova and Soffel (2012)

[32] Of course, the possibility that the centre line of foundations was oriented first cannot be excluded.

astronomically contains only the errors of astronomical observations, the orientation of the sides of the pyramids adjusted to this direction by geometrical means depends also upon unknown errors in realization of parallel constructions and right angles. In the following discussion, we will consider first the accuracy of the orientation of the east sides (this set of data is also better known); the orientation of the west sides will be discussed later in the text.

The right angles at the corners of the pyramids could have been constructed in the process of the Stretching of the Cord ceremony with the help of a cord marked at intervals of 3, 4 and 5 units. On can hardly imagine, however, a cord with the length of the side of Khufu's pyramid;[33] probably, a small right-angled triangle was built up first at a corner and then the plane of the pyramid was staked off with the help of existing surveying methods. Orientations of the sides of the pyramids given in Table 1 permit calculation of the inner angles of the basement and estimation of the precision of their construction (see Appendix C).

In the following discussion, we will try to estimate the accuracy and precision of the methods based upon the stellar alignments recently proposed and discussed in the scientific literature. These proposals are all based upon Assumption 1 and the solutions aimed at explaining the apparent trend in the orientation of the pyramids can be *per se* considered as a justification of this assumption. We should not forget, however, that if we rejected this assumption, we would still face the problem of the orientation of the pyramids upon cardinal directions and would either consider it as fortuitous or propose a method which could support the observed accuracy of their orientation.

2 Alignments and Errors

Many notable stellar configurations drew the attention of the observers in ancient Egypt and obtained their own names; some of these configurations are identified, but the names of others are still under debate. Does the available information permit us to draw any firm conclusion about which configurations of stars were used to orient the ancient monuments?

The simplest noticeable geometrical forms built up by stars are their linear alignments, e.g. vertical and horizontal arrangements. Astronomical observations of both variants could be realized with simple instruments following natural physical laws: verticals with plumb lines, and horizontal lines with the help of water levelling. The idea of using a vertical alignment of stars in the orientation of pyramids seems nowadays very natural: it follows from what we have already written about finding the true north direction for an observer at a given position on Earth—the north point at the horizon is defined as a point where the celestial meridian running through the zenith of the observer and the North Pole crosses the horizon under a right angle. To attribute such considerations to the ancient surveyors would be, however, anachronistic; some other ideas might have played an important role—e.g. a vertical alignment of stars as a sort of a stellar elevator to the celestial realm (if so, the stellar vertical would not need to be observed near the meridian). It is also conceivable that a specific arrangement of stars could have served to fix a moment of time to target a certain star considered to be the final destination of the celestial voyage.

It is important to understand that there exist many stellar positions which can explain the orientation of *any single* pyramid by modifying the year of its foundation. But to propose a new date for a single pyramid would not explain the orientation of *all* the pyramids: it is the main trendline of their azimuths which serves as a constraint to the possible solutions. And the gradient of this trendline strongly depends upon the lengths of reign and is, therefore, connected with a certain relative chronology. A very important aspect of the problem is the presence of

[33] According to Badawy (1957, 57), the Egyptian word for 'to found' and for 'plan' was written with a looped cord as a determinative. Dorner (1981, 9) assumes that the cords used for surveying would have been suitable only for rough measurements because the total length of the cord would have been influenced by atmospheric humidity.

two trendlines in the orientation of the pyramids. No doubt that the procedure was essentially a religious matter, carried out according to strict rituals at fixed dates, but the pattern of orientation of the pyramids displayed in Fig. 1 makes it also evident that either the same stellar configuration (or a single star) was the target at *different* moments of time[34] or it was not always the same target object which was used in the alignment of the pyramids.

It is well known that there is a limit to the precision of the resolution of the human eye,[35] but a major limitation to the angular resolution in astronomical observations is the effect of *seeing* which includes the degradation of the stellar image due to turbulent airflows in the Earth's atmosphere.[36] The best possible precision of naked-eyed observations was achieved by Tycho Brahe: the mean error in Brahe's final published catalog was of around 2', with a systematic error of as much as 3' in some of his stellar positions (Rawlins 1993). As is well known, Brahe designed and used the larger versions of the sextant and the quadrant and, without suitable instruments, Tycho Brahe's precision cannot be attributed to the surveyors of ancient Egypt who made their observations with the naked eye and without sophisticated technical tools.

What kinds of errors might have been involved in the orientation of the pyramids towards a certain stellar configuration? Obviously, there are some technical problems of the methods which can be divided into the following groups: a) realization of a vertical or horizontal line, b) targeting the stars by using a suitable device, c) drawing a line on the ground which gives the direction towards the star.

The vertical alignment of stars can be observed along a plumb line—this is possibly the most precise part of the algorithm; a horizontal alignment can be surveyed along a simple water-levelled surface of a wall where the positions of the stars can be marked. The level can also be established geometrically, i.e. perpendicular to the direction given by the plumb line. However, the bigger the angular distance between the stars, the larger should be the construction used for observations. Certainly, one can place a device close to the observer; in this case, it will be smaller but the effect of parallax acts detrimentally here—it is easy to observe this by looking at a distant object relative to a finger of the stretched hand first with one eye and then with another. Parallax leads in this case to a visible jump of an object of about 5–8°; to eliminate this effect, one can agree to use either the left or right eye. The later textual evidence concerning the star clocks shows that such a convention was possible.[37]

Provided that we have observed two stars aligned in a certain way, the biggest problem left is to project the corresponding direction onto the local plane. In our opinion, this part of the procedure is largely responsible for the errors of its realization. To draw a line, we need at least two points on the ground. The first point is evidently given by projection of the plumb line used in observations (by targeting vertical alignments of stars) or by the projection of the mark on a wall (by horizontal alignments of stars). The position of the second point depends heavily upon the mode of sighting the object. For example, with the agreement to observe the alignment with one eye, one should fix the corresponding point on the ground by constructing a vertical going exactly through the pupil of this eye. One can improve the precision of the procedure by observing the stellar alignments with the help of *two* constructions, both levelled correspondingly. Generally, the greater the distance between the two marks on the ground, the

[34] In this case, both trendlines would have similar gradients due to the same values of right ascensions and declinations of stars/star. The small difference in gradients shown in Fig. 1 cannot, however, be used to finally reject this possibility because of sparse data for the second trendline.

[35] According to Rayleigh's Criterion it attains about 1 arcminute at daytime. Depending upon the lighting condition and the observed source, the resolving power of the human eye was estimated as 3 arcmin (Herrmann 1975; Gribbin, and Gribbin 1996).

[36] This effect is especially pronounced in deserts where it is caused by a significant difference between the sand's and atmospheric temperature at night.

[37] Let us recall that in the Ramesside star clocks the stellar positions were described in relation to the parts of the observer's body (e.g. by the left/right shoulder, left/right ear, left/right eye, opposite the heart).

smaller the error in drawing the direction line—thus, it is preferable to place the second device at some distance from the first.

However, to observe a vertical alignment of stars covered by *two* thin threads placed at a noticeable distance without a sort of sight-seeing tube is not possible. An alternative solution would be to observe such alignment with the help of the stable vertical constructions, with the sides levelled with plumb lines; in the simplest case, with two poles mounted vertically. With a high probability, this was the method used in the Stretching of the Cord ceremony. The large construction, however, should be stable and have a certain breadth. To give an idea: to observe a star at the altitude of 45°, the height of the vertical pole should be equal to the distance to the pole being used. Let us take as an example a pole with a breadth of 10 cm and a height equal to 7.2 m. It will cover the necessary altitude from the distance of 7.2 m[38] where the observable breadth of the pole attains 48´. Obviously, one cannot observe a star *behind* the pole with non-zero breadth and one will be forced to observe the target star either along the eastern or western side of the pole. The direction line in this case should be drawn on the ground as a line connecting the corresponding edges of the two poles. But did ancient architects always do that or did they accidentally take a middle point at the base of a pole to draw a direction line?

The alternative solution discussed in the scientific literature—to observe the stars through the rifts of two *bays* both mounted vertically—is difficult to realize because the sighting is possible in practice only at a very small mutual distance between two *bays*. We would have the same problem with the *merkhet*: the *merkhet* can be used as a handy instrument to approximately estimate the vertical alignment of two stars,[39] but to draw a line on the ground by observing a star with two *merkhets* at some distance is not possible.

With the technical problems being solved, the mathematical problem of treating the observations becomes of primary importance.[40] It is well known that Tycho Brahe paid attention to careful treatment of the observations. According to W. G. Wesley, he was "the first to see that it is also necessary to take long series of observations so that random, instrumental and human error can be averaged out" (Wesley 1978, 52). Let us notice that this practice was alien even to Claudius Ptolemy.[41] The ancient architects probably used for the orientation of the pyramids a single observation which they considered on some grounds to be the best, and it is highly improbable that the direction line was obtained as a mean result of a series of observations.

To summarize, we assume that the realization of the direction towards a specific stellar configuration at the local plane has always contained different kinds of systematic errors and that for the stars at high altitudes, the accuracy would had been significantly worse than for the stars at lower altitudes.

3 Vertical Alignment of Stars

The method of orientation proposed by Kate Spence in 2000 was that "the pole was considered to be located on an invisible chord linking two circumpolar stars on opposite sides of the pole. These two stars rotate around the pole, and when they are vertically aligned above the north horizon (one at its upper culmination and another at its lower) an alignment made towards these stars with a plumb line will be exactly oriented to true north, as long as the chord itself passes

[38] The distance corresponds to the hyperfocal distance at night which is, at its simplest, the focusing distance that gives us the greatest depth of field. This distance allows observation of both the device (in this case, the pole) and the distant star as sharp objects.

[39] The *merkhet* could also be used to observe stars at their horizontal alignment along a handle with the hour marks.

[40] On the history of statistical methods in astronomy see Sheynin (1993, 153–192).

[41] According to G. Graßhoff, Ptolemy "had no access to a method of finding the mean value" (Graßhoff 1990, 211). The least squares method was introduced to scientific practice by Gauss and Laplace.

precisely through the pole" (Spence 2000, 322). The author apparently presumed that the concept of the North Celestial Pole must have existed at the time of the construction of the pyramids and that their sides were oriented according to this epistemological concept—but this is in no way obvious.[42]

The simultaneous culmination of a pair of bright stars is a rare event. The equatorial coordinates of stars—the right ascension α and the declination δ—change with time due to the effects of precession and nutation and due to the proper motion of stars. As a result, the stars which have once culminated simultaneously will not do that with time but will still be aligned vertically twice during 24 hours—once to the west and once to the east of the meridian with approximately the same angular distance to it. The latitude of the location and the coordinates of stars are responsible for creating the possible seasons of observation. For the latitude of Giza at the time of construction of the Great Pyramids, the pair of stars proposed by K. Spence—Mizar (ζ Ursae Majoris) and Kochab (β Ursae Minoris)—could have been observed with Mizar at lower position (and Kochab at the upper) in summer and, reciprocally, with Mizar at the upper position (and Kochab at the lower) in winter. A problem specific to Spence's stars is as follows: for the pair Kochab and Mizar one should observe an arc of ca. 42° above the horizon.[43]

Let us now estimate the errors in the possible realization of this method. Astronomical modelling of the 'simultaneous transit method' was included in the original paper of K. Spence with a mathematical error in calculation of the azimuths noticed by D. Rawlins and K. Pickering (2001, 699). For an observer at a known latitude, a pair of stars is aligned vertically if they are observed at the same azimuth; provided that the equatorial coordinates (α_1, δ_1) and (α_2, δ_2) of two stars are known for a selected date, formulae given in Appendix A permit a correct calculation of the azimuth of their vertical alignment. To visualize the vertical alignments of stars K. Spence used the astronomical software SkyMap Pro6; recent comparison of astronomical software currently available for archaeoastronomical applications demonstrated, however, noticeable differences between the programs and following the recommendation of De Lorenzis and Orofino (2018), the freely available software *Cartes du Ciel* based upon the latest precession theory (Vondrák et al. 2011) will be used throughout this paper.[44]

The vertical alignment of the two stars as it was seen at the latitude of Khufu's pyramid at 14.01.2478 BC is displayed in Fig. 2.

[42] The idea to use a vertical alignment of stars was perhaps proposed first by Polák (1952, 219–220). Spence called her method 'the simultaneous transit method'; we prefer to refer to it as 'the method of vertical alignment of stars', because 'transit' is reserved in astronomy for the meridian transit of luminaries.
[43] K. Spence proposed a sort of giant *merkhet* with a plumb on a long string. Such a construction is not documented in the ancient Egyptian sources.
[44] Parkerson (2019). Note that the nutation is neglected in the program for the time period before 500 BC. The nutation series do not contain by definition secular terms and should contribute to our calculations below the level of 0.5´. Another unknown constraint of the ancient observations—the effects of refraction—does not noticeably influence the azimuths of the vertical alignments of stars: refraction mainly acts by 'raising' an observed object.

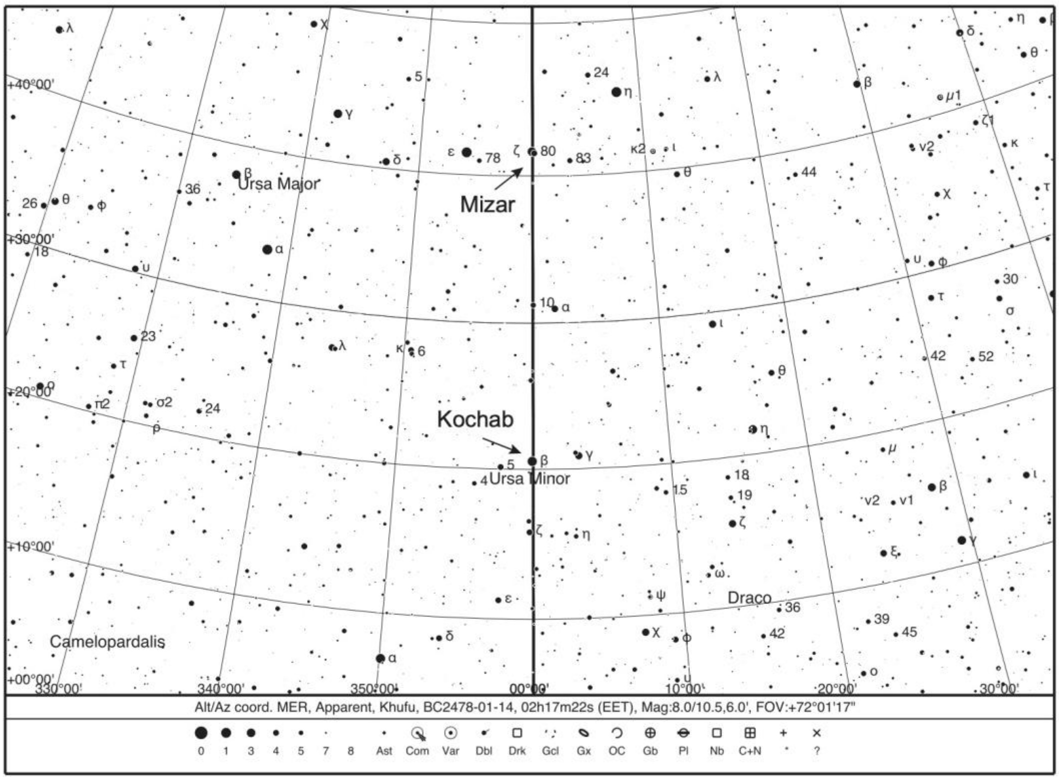

Fig. 2: Vertical alignment of Mizar and Kochab at the latitude of Khufu's pyramid on January 14, 2478 BC. Map produced on Cartes du Ciel.

Azimuths of the east sides of the pyramids[45] vs. azimuths of the vertical through Mizar and Kochab given in Table 2 are shown in Fig. 3 for the dates used by K. Spence (von Beckerath's chronology given in Table 2, col. 5 shifted by 46 years to account for Snofru's reign, that is, assuming 2600 BC for Snofru's accession date).[46] Firstly, it is evident that there is a typing error in the text of Spence (2000, 321). In her Fig. 1 (b) the line 'a' must correspond not to the position of the star Kochab in UMi at the upper position as the author states, but to the star Mizar at the upper and Kochab at the lower position. This changes the season of possible alignment: it took place around winter instead of summer time. In Fig. 3, the line 'a' represents correctly the azimuths of the alignment of Mizar at the upper position with Kochab at the lower position. The opposite alignment (line 'b') shows a distinctly different trend in azimuths and should be excluded as a possible target. The gradients of the trendlines depend upon the orientation of every single pyramid, and could of course change with a different new measurement. However, it is important to understand that the trendlines do only approximate *linearly* the complicated curves representing the azimuths of the vertical alignments which change over time due to the effects of precession; according to our estimation, the difference between this linear approximation and the true azimuth can attain more than 1´ over 200 and more than 3´over 500 years. Another source of approximation is due to the different geographical positions of the pyramids: in fact, the gradients of the trendlines will slightly differ for every pyramid. Thus, the correct approach to estimating the precision of the alignment of the pyramids would be to calculate the deviations between the azimuths of the sides of the

[45] Djoser's pyramid seems not to be aligned in this way and will not be displayed in the following diagrams.
[46] The equatorial coordinates of the stars proposed by K. Spence change only slightly during one year and do not greatly influence the following results over this time span.

pyramids and the azimuths of the target stellar configuration for every pyramid separately at the assumed date of its foundation.

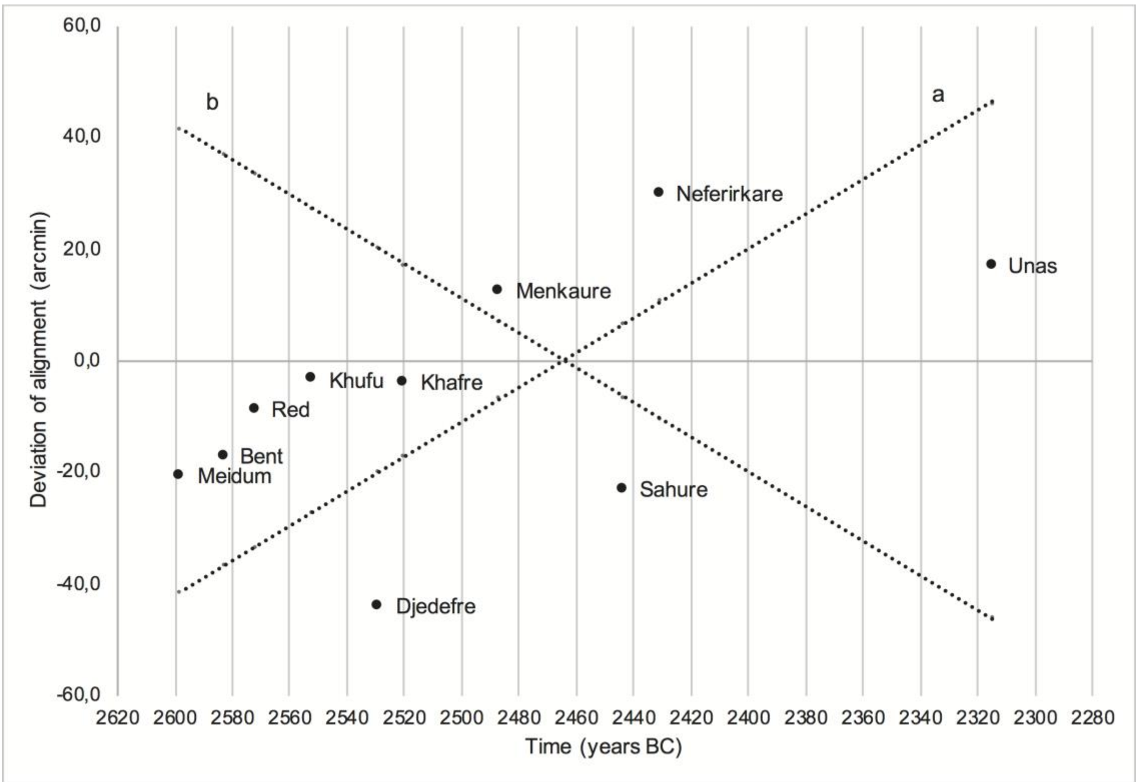

Fig. 3: Deviation of pyramid orientations (east sides) from true north over time vs. azimuth of the vertical alignment of Kochab and Mizar (von Beckerath's chronology using 46 years for the duration of Snofru's reign). The dashed line 'a' is a trendline through the points corresponding to the azimuth of the vertical alignment of Mizar at the upper and Kochab at the lower position. The trendline 'b' corresponds to Mizar at the lower and Kochab at the upper position.

These deviations are given with respect to the time for the trendline 'a' in Fig. 4. Here, a zero-value on the y-axis corresponds to a perfect orientation and the deviations are calculated as dA - DA where dA are the azimuths of the east sides of the pyramids (Table 1, col. 6) and DA are the azimuths of the vertical alignment at the corresponding place and assumed time of the foundation ceremony. Two patterns are clearly recognizable: three pyramids, Djedefre's, Sahure's, and Unas' have negative errors between -24´ and -30´ whereas the Meidum, Bent, and Red pyramids as well as Khufu's, Menkaure's and Neferirkare's pyramids show smaller positive errors in the orientation towards the vertical through Kochab and Mizar. Although the number of pyramids does not allow a proper statistical analysis, with some degree of confidence we can estimate the mean value of deviation of the orientation of the latter group as 21.2 arcmin with a standard deviation of 2.5 arcmin. The last value points towards a very small random error, that is, towards an astonishingly *precise* adjustment procedure.

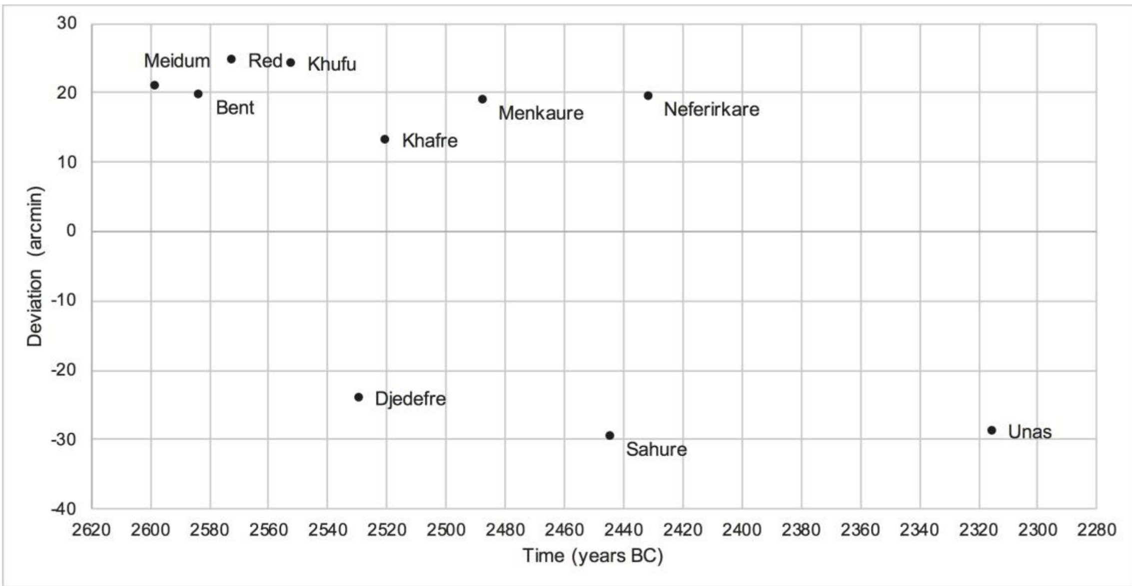

Fig. 4: Precision of pyramid orientations (east sides) towards the vertical alignment of Mizar at the upper and Kochab at the lower position (von Beckerath's chronology using 46 years for the duration of Snofru's reign).

This observation partly revises a critique of Spence's method put forward by Nell and Ruggles :

> A question left unanswered regarding the 'simultaneous transit' strategy for determining the north direction is on what grounds the ancient Egyptians made a choice of a particular pair of stars. For example, the choice of Kochab and Mizar would, as Spence has shown, have achieved the greatest precision at the time of Khufu and Khafre, but it is inconceivable that these stars were chosen for this reason some 50 years earlier; instead, we would have to assume that the level of precision achieved by those pharaohs was fortuitous. If, on the other hand, the level of precision achieved at Gyza was deliberate rather than fortuitous, then the surveyors must already have determined that a particular pair of stars, when aligned vertically in the sky, would provide a particularly good indication of true north (Nell and Ruggles 2014, 308).

The authors seem to have mistaken errors in the alignment to true north for the errors in orientation towards the stellar vertical: as seen at Fig. 4, the deviations from the orientation of the pyramids towards this vertical do not show a secular trend over time and remain at approximately the same level.

It is worth noticing that after the proper calculation of the azimuths of the stellar vertical, Khufu's pyramid is no longer the best oriented towards this target, although it is best oriented towards true north. In fact, it is the pyramid of Khafre which seems now to be best oriented towards the stellar vertical proposed by K. Spence. The known problem of its orientation, however, is that it is identical with that of Khufu's pyramid in spite of more than 30 years between the accession dates of the two kings. It is not only practically the same orientation of the sides of these two pyramids as shown in Table 1, but also the descending passages have a strikingly identical orientation: -5.6´ vs -5.8´, respectively (Petrie 1883, 58, 104). The proposed hypotheses to explain this are a) copying of alignment,[47] b) two pyramids laid down

[47] Rawlins 2003: 3.

simultaneously (Khufu's double project),[48] or c) change of the position of Khufu's pyramid.[49] That is why the data for Khafre's pyramid, although shown in the graphs in this text, will not be used in the calculation of trendlines, mean values or the standard deviations of data.

As shown in Fig. 5, the trendline 'a' through the azimuths of the vertical alignment of Kochab and Mizar has a gradient similar to the gradient of the trendline 'c' through the azimuths of the east sides of the pyramids, -0.31 vs. -0.2943,[50] and that was the crucial argument of K. Spence for the choice of this pair of stars.

The deviations between the three lines—'a', 'c', and 'd'—can be interpreted in different ways. First, as a systematic error of the method in the range of about ±25'. Such an accuracy, close to the apparent size of the Sun, is what we would expect from all we know about the surveying techniques in ancient Egypt; however, a counter example is provided by the 4th Dynasty pyramids. It is the extraordinary precision of their alignment to true north which forces us to choose a second interpretation and to infer that the method of the orientation of the pyramids had a very small systematic error—this interpretation was taken by K. Spence, followed by many other scholars.[51] K. Spence had possibly not calculated the precision of the orientation towards the vertical alignment of Kochab and Mizar, and proceeded in the following way. Assuming that the chronology for line 'a' in Fig. 5 can be considered fixed, "the chronology according to which the archaeological data are plotted [line 'c' in our Fig. 6] is not anchored in time. However, the point at which line ['a' in our Fig. 5] crosses zero on the y-axis can now be fixed at 2467 BC from the results of the astronomical modelling. This gives a date of 2478 BC for the alignment of Khufu's pyramid which would require the lowering of von Beckerath's lower estimate of chronology by a further 74 years" (Spence 2000, 324).[52] Thus, the difference between the points where the two lines—'a' and 'c'—crosses the line y = 0 was used as a temporal shift to align these two lines.

[48] Shaltout, Belmonte, and Fekri 2007; Magli 2005, 2009; Magli and Belmonte 2009.

[49] This idea was proposed by Kruglyakov (2016, unpublished) and discussed by Puchkov (2019, 49). For the chronology of the pyramids' alignments, the difference between the casing and core azimuths of Khufu's pyramid (Petrie 1883, 126) can be of great importance.

[50] The gradient is negative because of the chosen orientation of the x-axis (time in years BC). Note the slight difference in the gradients of lines 'c' in Fig. 1 and Fig. 5 caused by only two years difference in the date of the alignment ceremony for the Meidum pyramid.

[51] Rawlins and Pickering 2001; Belmonte 2001; Magli 2003; Puchkov 2019. Many attempts to correct the existing Old Kingdom chronology with the help of astronomical phenomena completely ignore the problem of systematic errors and can therefore lead to ambiguous results.

[52] According to our calculations, the line 'a' crosses the zero line of the y-axis at 2464 BC.

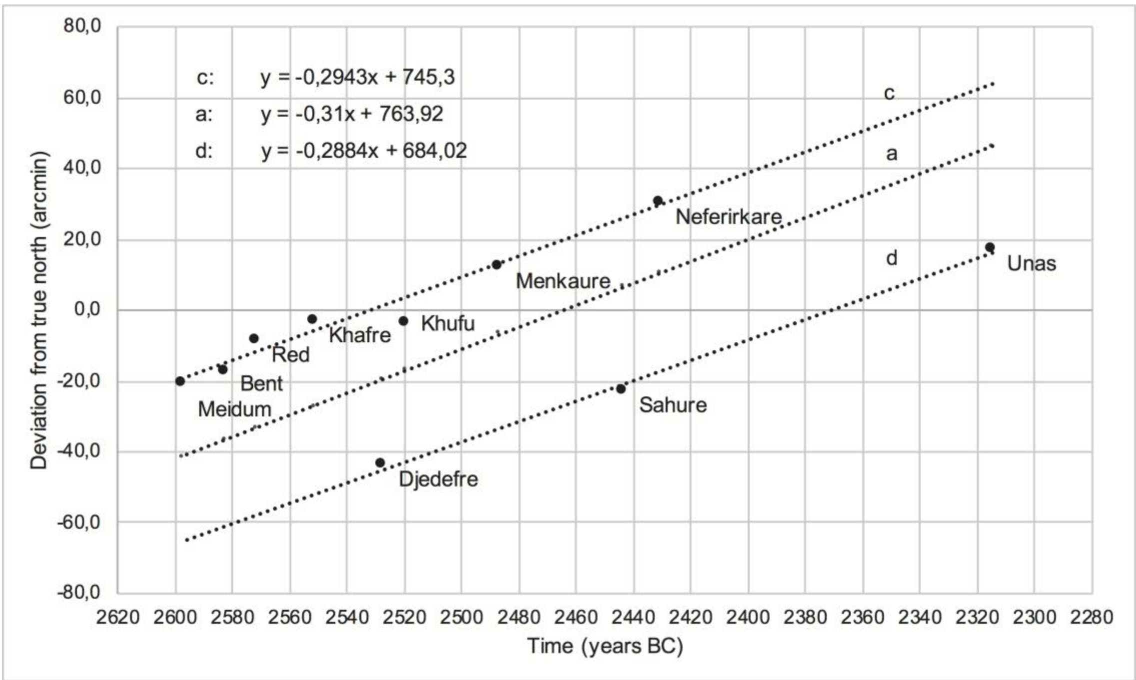

Fig. 5: Deviation of pyramid orientations (east sides) from true north over time vs. azimuth of the vertical alignment of Mizar and Kochab (von Beckerath's chronology using 46 years for the duration of Snofru's reign). The dashed line 'a' is a trendline through the points corresponding to the azimuth of the vertical alignment of Mizar at the upper with Kochab at the lower position.

Fig. 6 displays the results of this chronological correction. The errors in the orientation of the east sides of the pyramids towards the vertical alignment of stars are now calculated for the dates of alignments minus 74 years as proposed by Spence (2000). The mean value of the deviations for the alignment of the pyramids along the main trendline becomes now -2.1´ but the standard deviation from the mean value remains approximately the same (2.3´) because it characterizes the same precision of the method.[53] In contrast, the deviations in the orientation of three other pyramids—Djedefre's, Sahure's and Unas's—are now much larger and can hardly be attributed to the same method. K. Spence tried to solve the problem of the orientation of Sahure's and Khafre's pyramids by assuming that they were aligned at a different season, at the moment of the vertical alignment of Mizar at the lower and Kochab at the upper position (Fig. 3, line 'b'). That would change the strict ritual rules, but the alternative solution—that a different star or different stellar configuration was used to align Djedefre's, Sahure's and Unas's pyramids—would also require them to have been changed.

[53] The small difference from the previous standard deviation is due to proper motion of stars accumulated in 74 years.

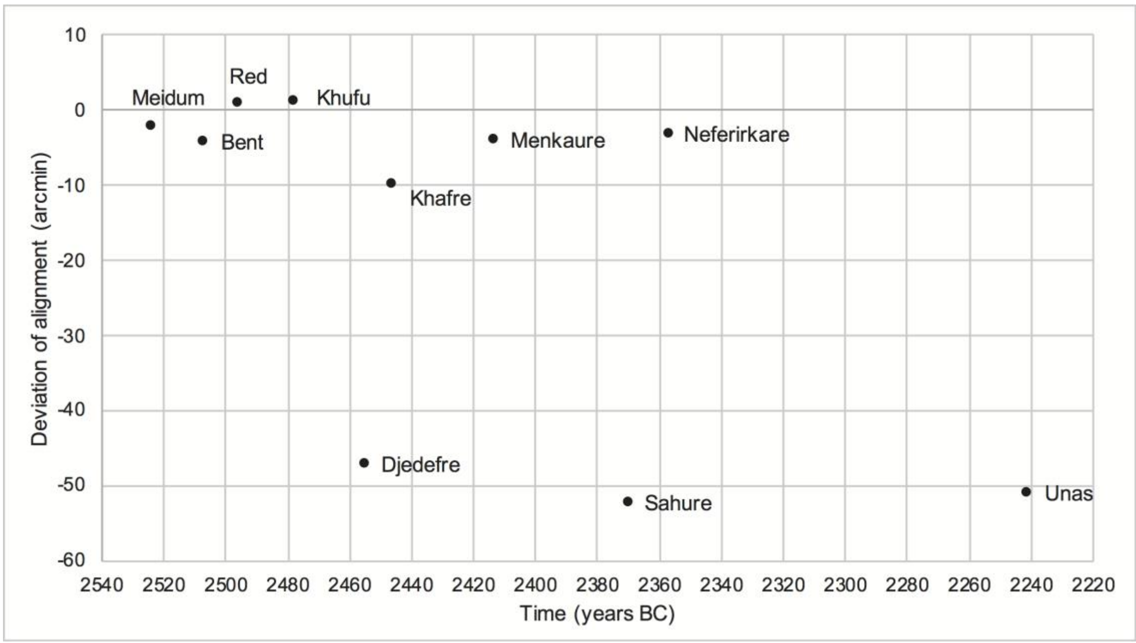

Fig. 6: Precision of pyramid orientations (east sides) towards the vertical alignment of Mizar at the upper and Kochab at the lower position (Spence's modified chronology).

The chronological correction proposed by Spence (2000) is not unproblematic. It was not specified in which way the line 'c' was constructed; possibly, with a standard program minimizing the mean squared errors of all the digressions, including the complicated case of Khufu's pyramid. A possible source of ambiguity is also involved in drawing a chronological line; Spence (2000) assumed that the pyramids were *all* aligned in the second year of the reign. If not, the trendline will have another gradient, followed by another shift in chronology, greater than the five years of confidence interval given by Spence. It is worth mentioning that the deviation between the trendlines can also be interpreted as a time delay in the alignment procedure.[54] In the following text we will, however, refrain from this unorthodox interpretation and assume that the alignment of pyramids was performed at the moment of a specific sacral configuration of stars.

The starting chronology for Spence's algorithm was that of von Beckerath with 46 years for Snofru's reign. The same procedure of 'chronological correction' can also be applied to the dates of the alignment ceremonies corresponding to Show's chronology. However, in this case the solution will become more ambiguous, because the deviations of Djedefre's, Sahure's, and Unas' pyramids as shown in Fig. 7 will be smaller than those of the pyramids along the main trendline and it would be more natural to base the chronological correction upon the data for this group, thus leaving aside the challenging question of the orientation of the pyramids of the 4th Dynasty.

[54] For example, let us assume that—for some kings—a ritual formula/spell for *Entering the Sky* should have been read starting at the moment of the vertical alignment of stars followed by marking out the direction towards Kochab. Then, a delay of about 9 minutes would fully explain the orientation of the pyramids along the main trendline 'c'.

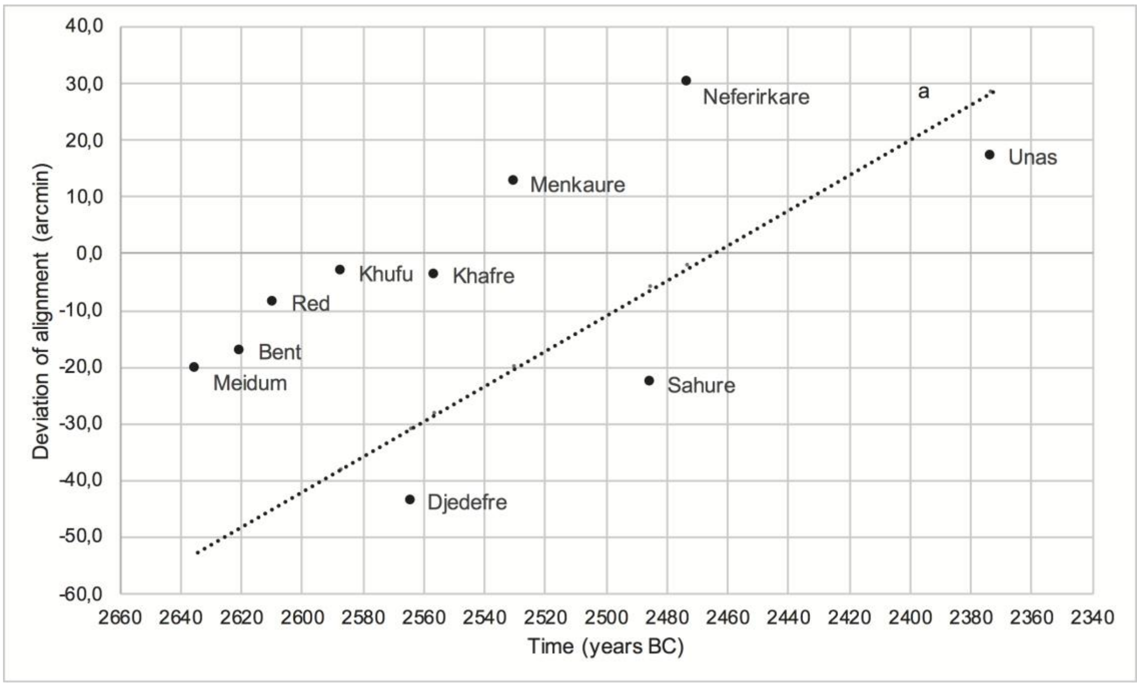

Fig. 7: Deviation of pyramid orientations (east sides) from true north over time vs. azimuth of the vertical alignment of Kochab and Mizar (Shaw's chronology using 46 years for the duration of Snofru's reign). The dashed line 'a' is a trendline through the points corresponding to the azimuth of the vertical alignment of Mizar at the upper with Kochab at the lower position.

To reformulate more precisely the question put forward by Nell and Ruggles (2014, 308): on what grounds did the ancient Egyptians conclude that a chord passing through a particular pair of stars would also pass through the Celestial Pole?

The ambiguity of the solution becomes even more evident if one poses a simple question: why *those* two stars?

An alternative pair of stars—Phecda (γ Ursae Majoris) and Megrez (δ Ursae Majoris)—was proposed by J. A. Belmonte (2001). In contrast to the stars chosen by Spence (2000), these stars are aligned vertically, both being close to the lower or upper culmination. The requirement that one of the stars is at its upper and another at the lower position is not mandatory. If not adhered to, it would mean, however, that the stars would be moving in the same direction (in the western direction at the upper culmination and in the eastern direction at the lower culmination) and the *perceived* duration of their vertical alignment lasts longer. The observational situation at Giza in 2571 BC is shown in Fig. 8.

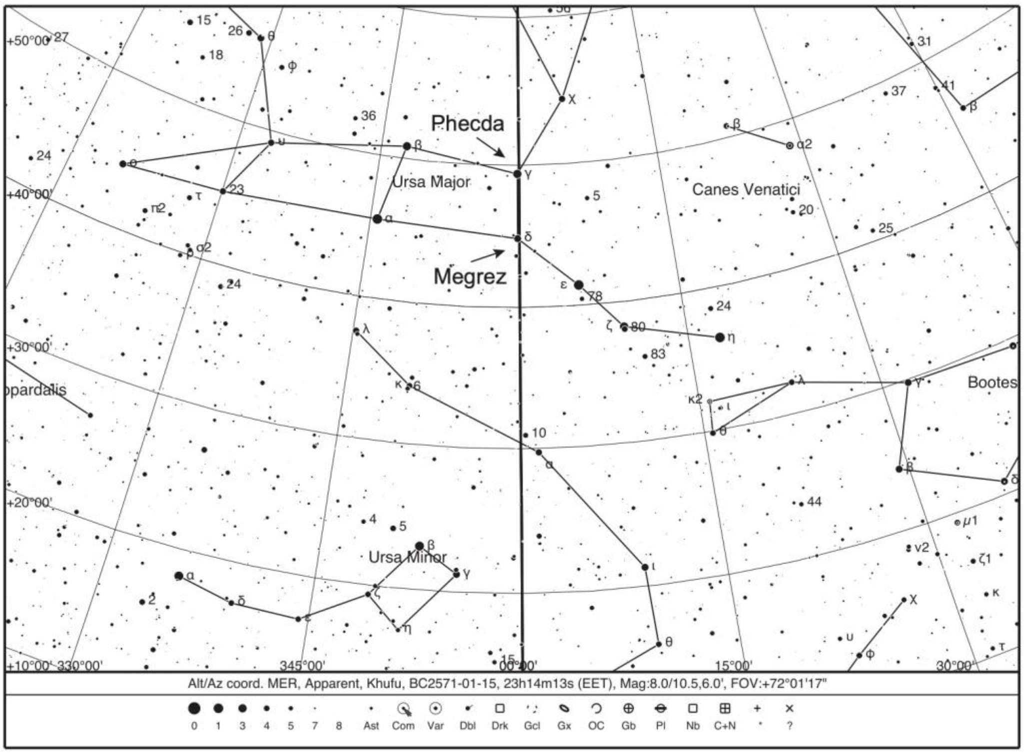

Fig. 8: Vertical upper alignment of Phecda and Megrez at the latitude of Khufu's pyramid on January 15, 2571 BC. Map produced on *Cartes du Ciel*.

J. A. Belmonte based his method upon the same assumption as K. Spence: that the real goal of the pyramid orientation was the North Celestial Pole. According to this author, Khufu's pyramid could have been aligned between 2571 and 2565 BC, at the time when Phecda and Megrez were at their upper vertical alignment; that gives "dates for Khufu's ascension to the throne that are just in the middle of the highest and lowest chronologies accepted today by Egyptologists"(Belmonte 2001, S12).[55]

With the actual precession theory, we can state that for the location of Khufu's pyramid, the vertical through Phecda and Megrez crossed the meridian around 2552–2551 BC.[56] We will now check Belmonte's proposal for three chronologies discussed in this text. For Shaw's chronology, the deviations of the orientation of the pyramids from true north (trendlines 'c' and 'd') vs. azimuths of the vertical through Phecda and Megrez (trendline 'a') are plotted in Fig. 9.

[55] The author considers in his text the high chronology according to J. Malek (Shaw 2000, 89–117) who assumes for Khufu's reign 2589–2566 BC and low chronology according to Baines and Malek (1980, 36) with the dates of 2551-2528 BC. Belmonte (2001, S11) correctly rejected the vertical alignment of Phecda and Megrez at lower culmination, because "the pair of stars at lower culmination, …do not fit the chronological pattern accurately".

[56] A more accurate date cannot be calculated because of the absence of a nutation calculation for the time in question; according to our estimation, it might change the date of the vertical alignment in the range of ± 1 year.

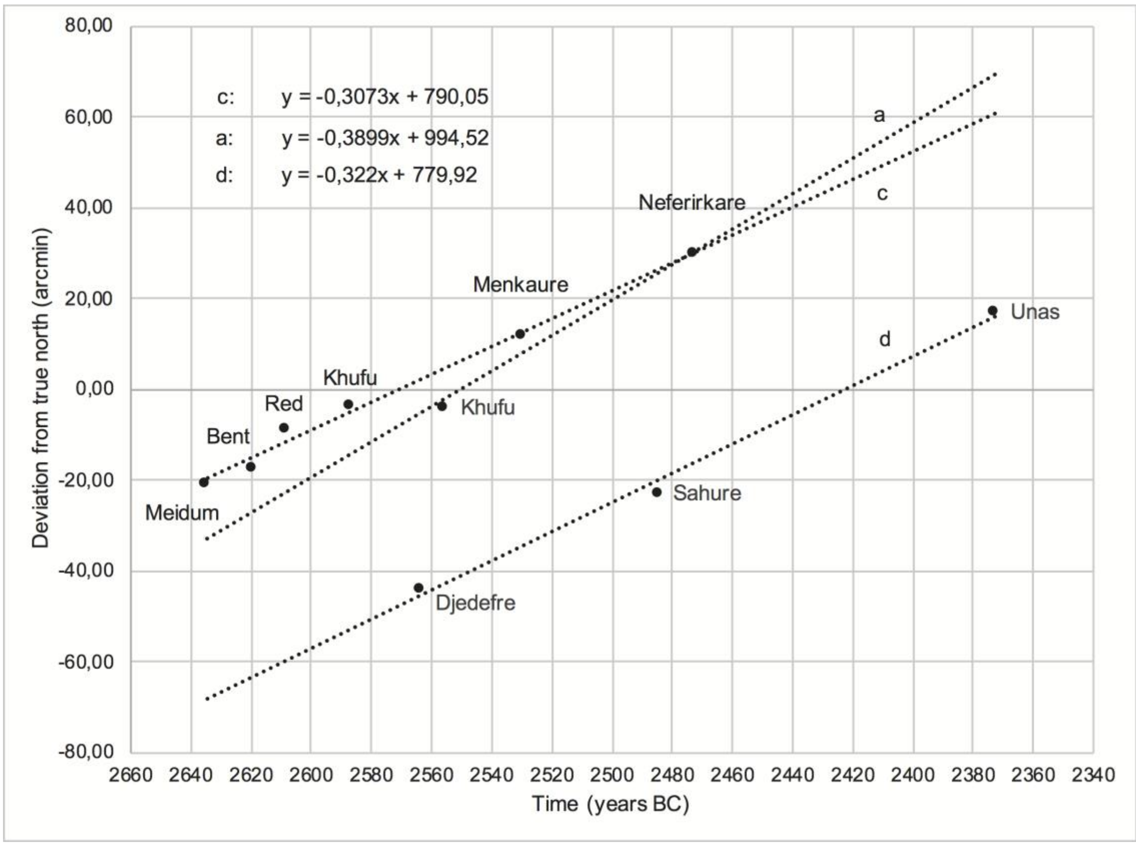

Fig. 9: Deviation of pyramid orientations (east sides) from true north over time vs. azimuth of the vertical alignment of Phecda and Megrez (Shaw's chronology). The dashed line 'a' is a trendline through the points corresponding to the azimuth of the upper vertical alignment of Phecda and Megrez.

In more detail, the errors of the orientation of the pyramids towards the vertical alignment of Phecda and Megrez, calculated as dA – DA, are given in Fig. 10. For a data set corresponding to the main trendline, Belmonte's proposal leads to a mean error of 8.5´ with a standard deviation of 5.5´, and the best oriented pyramid becomes that of Neferirkare.

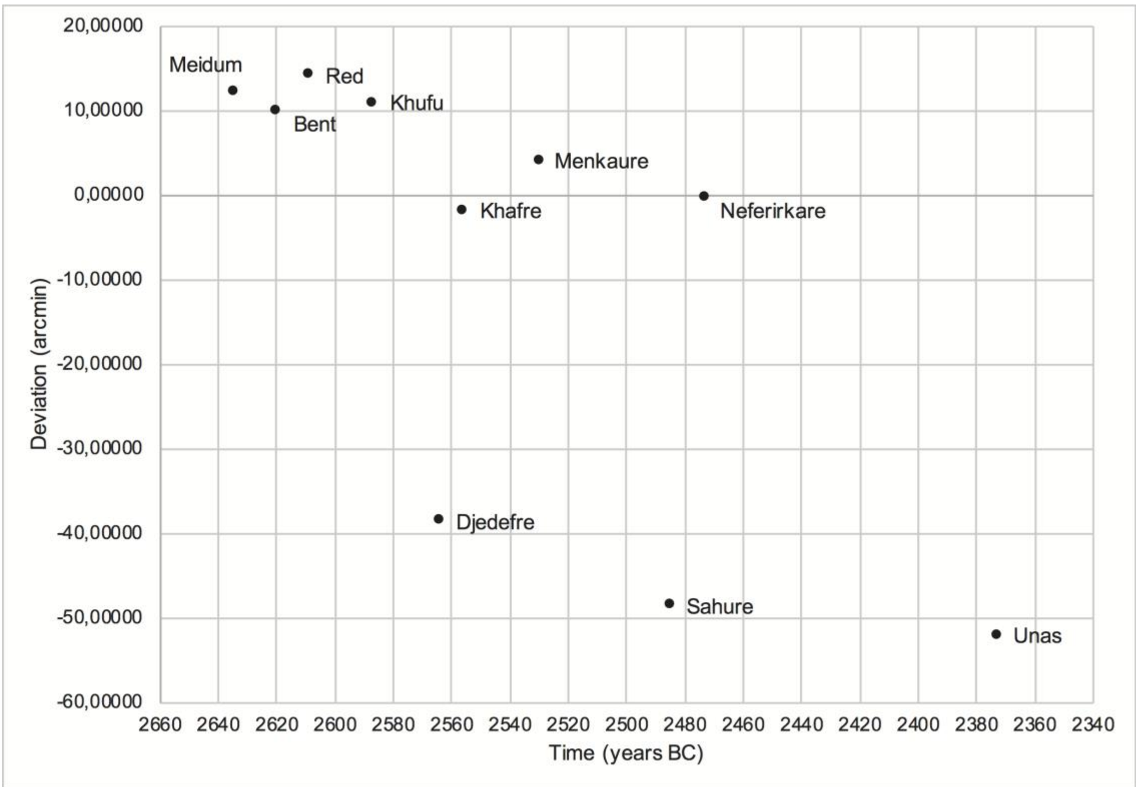

Fig. 10: Precision of pyramid orientations (east sides) towards the upper vertical alignment of Phecda and Megrez (Shaw's chronology).

We have also checked Belmonte's method for von Beckerath's chronology; the results are plotted in Fig. 11.[57]

[57] Note the slight difference in the gradients of the lines 'a' in Figs. 9 and 11; being a linear approximation to a complicated curve, a trendline depends sensitively upon the set of dates chosen to draw this line.

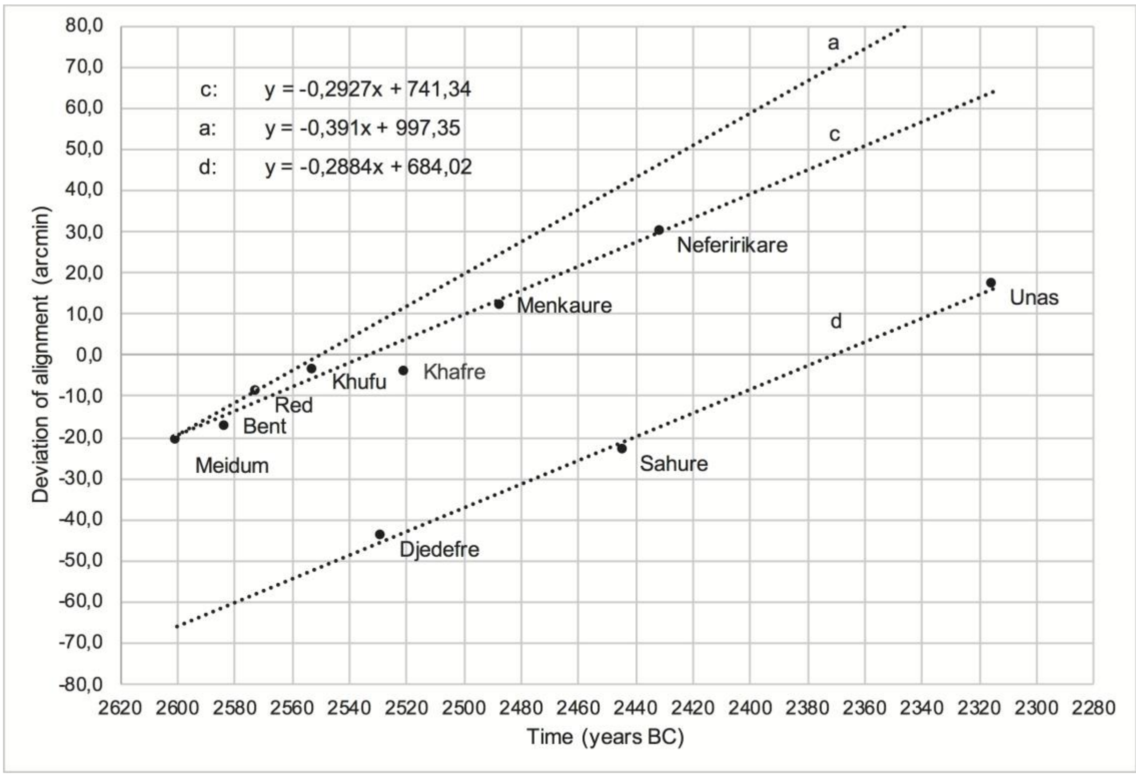

Fig. 11: Deviation of pyramid orientations (east sides) from true north over time vs. azimuth of the vertical alignment of Phecda and Megrez (von Beckerath's chronology). The dashed line 'a' is a trendline through the points corresponding to the azimuth of the upper vertical alignment of Phecda and Megrez.

Fig 12 shows the errors in the orientation of the pyramids towards the vertical alignment of Pecda and Megrez for von Beckerath's chronology.The mean error of the orientation of Snofru, Khufu, Menkaure and Neferirkare's pyramids now attains -6.5´ with a standard deviation of 6.6´.

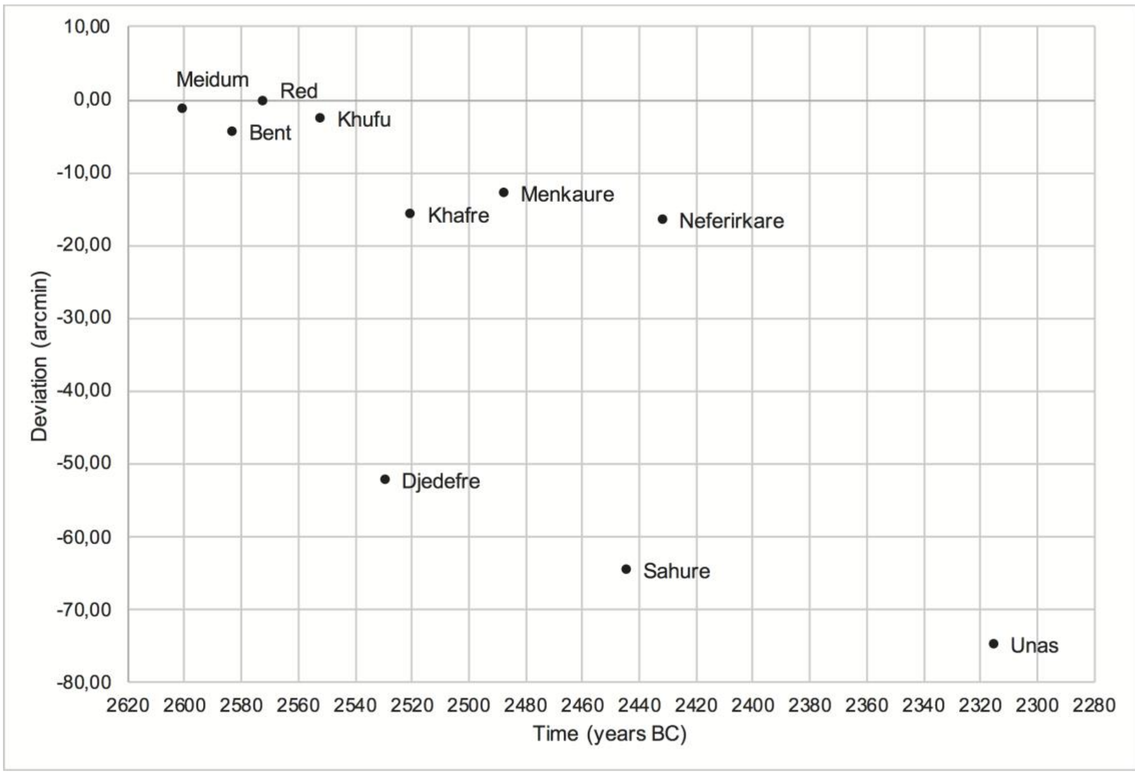

Fig. 12: Precision of pyramid orientations (east sides) towards the upper vertical alignment of Phecda and Megrez (von Beckerath's chronology).

The principal difference between the two chronologies for the pyramids along the main trendline becomes evident: for Shaw's chronology, the errors of the orientation are initially comparatively large but decrease over time—and this development seems to be logical—but that makes Neferirkare's pyramid be best aligned. To the contrary, for von Beckerath's chronology, the precision of the orientation is better for Snofru's pyramids and decreases steadily over time—on whatever grounds—and Neferirkare's pyramid becomes worst oriented towards the stellar vertical. Thus, in the first case we should conclude that the apparent precision of the northern orientation of Khufu's pyramid was purely fortuitous, and in the second case, that for unknown reasons the precision of the orientation towards the stellar alignment was decreasing over time.

The pyramids along the second trendline show for both chronologies large increasing errors in the orientation towards the stellar vertical (Figs. 9 and 11) and cannot be attributed to the same surveying method. One can assume that these pyramids were oriented towards a certain prominent star surveyed at the moment of the vertical alignment of Phecda and Megrez, but we were not able to detect any possible (reasonable) candidate. Belmonte (2001: S19) proposed alternatively that the pyramid of Unas might have been aligned to "the vertical transit of the handles of the Two Adzes, the stars Alkaid (η Uma) and Polaris (α Umi), with the former in lower culmination, in 2400$\pm$30 B.C." The azimuths of the vertical transit of Alkaid and Polaris vs. azimuths of the east sides of the pyramids are plotted in Fig. 13 where the trendline 'm' corresponds to Alkaid at the lower position and the trendline 'n' to Alkaid in the upper position. It appears that the first of these stellar arrangements can explain the orientation of Unas's pyramid but not the orientation of Sahure's and Djedefre's pyramids. To conclude, this proposal would require us to assume that the apparent trend in the orientation of these three pyramids was purely accidental.

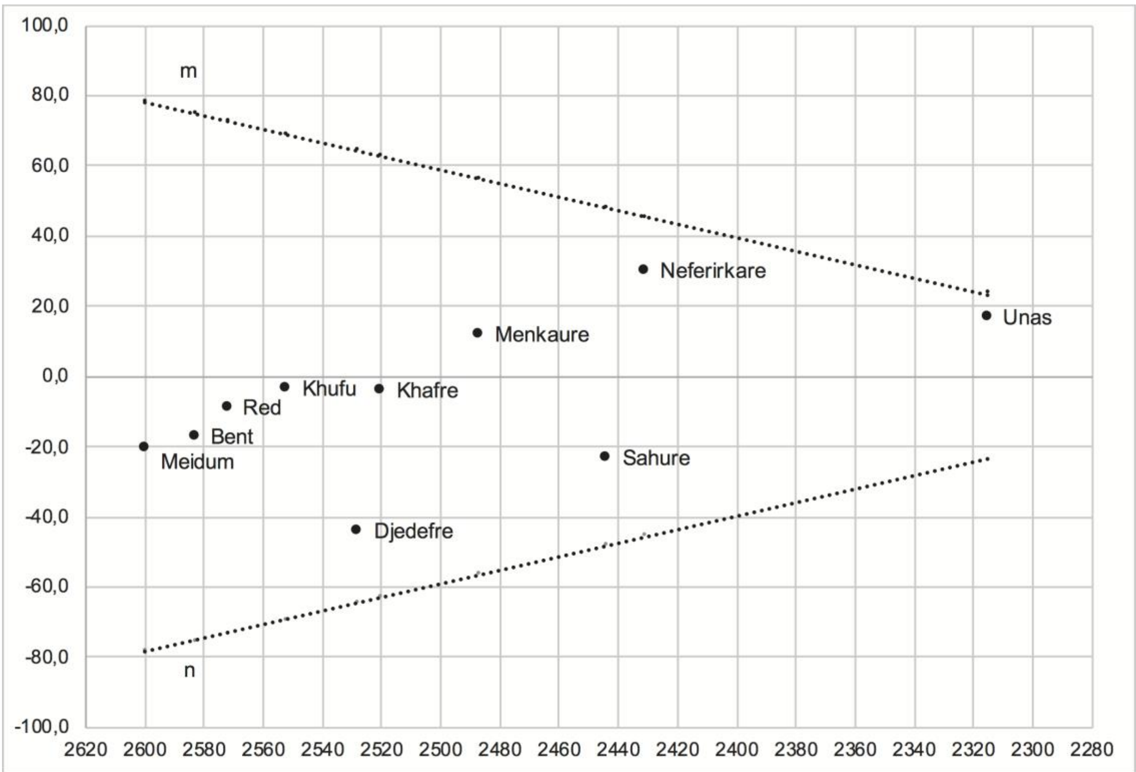

Fig. 13: Deviation of pyramid orientations (east sides) from true north over time vs. azimuth of the vertical alignment of Alkaid and Polaris (von Beckerath's chronology). The dashed line 'm' is a trendline through the points corresponding to the azimuth of the vertical alignment of Alkaid at the lower with Polaris at the upper position. The trendline 'n' corresponds to Alkaid at the upper and Polaris at the lower position.

Comparison of the results given in Figs. 9–10 and 11–12 shows that both chronologies reveal a secular trend in the errors of the orientation of the pyramids towards the vertical alignment of Phecda and Megrez and that the gradients of the trend line 'a' diverge from that of the trendline 'c' more than in the case of Spence's proposal (compare the gradients of lines 'a' and 'c' in Fig. 5 with Figs. 9 and 11).

We have also checked Belmonte's proposal vs. Hornung's chronology; the results given in Fig. 14 show that this chronology does not support the adjustment of the pyramids to the vertical alignment of Phecda and Megrez.

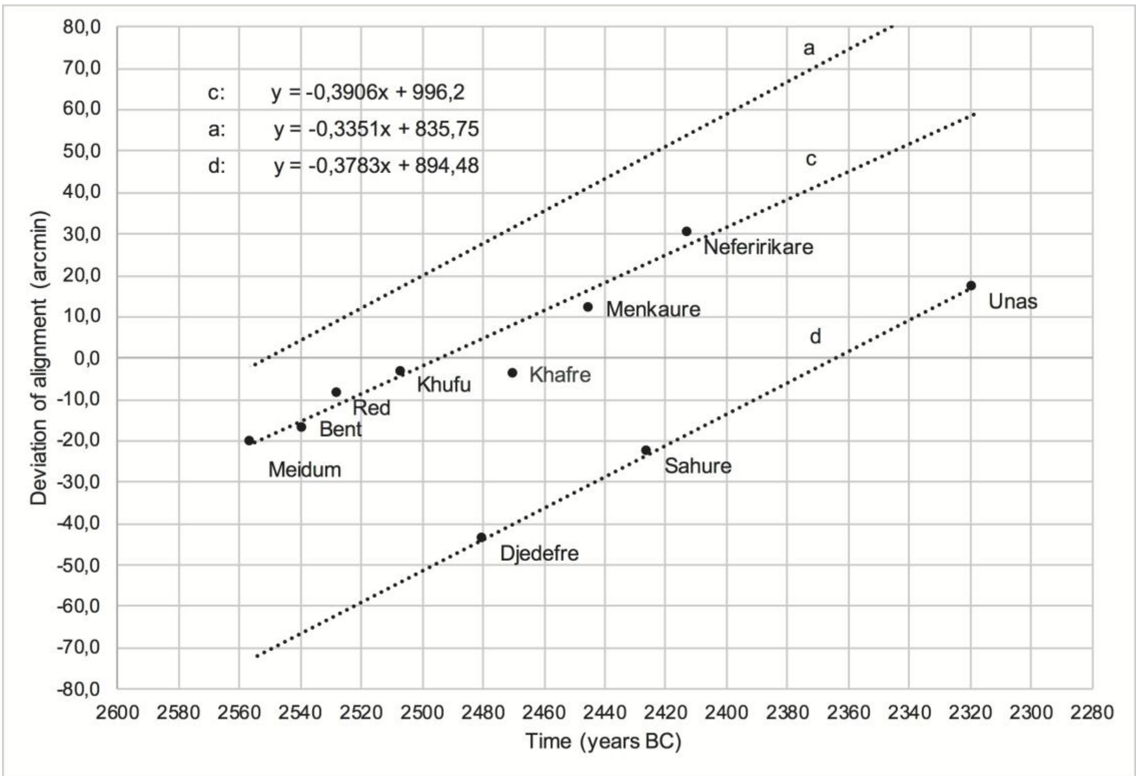

Fig. 14: Deviation of pyramid orientations (east sides) from true north over time vs. azimuth of the vertical alignment of Phecda and Megrez (Hornung's chronology). The dashed line 'a' is a trendline through the points corresponding to the azimuth of the upper vertical alignment of Phecda and Megrez.

The choice of Phecda and Megrez has the same problem as the pair proposed by K. Spence: to explain the trendline in the orientation of the pyramids, one should consider the vertical alignment of stars at their upper position, i.e. with Phecda at almost 50° and Megrez at about 45° above the horizon (see Fig. 8). To solve the problem, Belmonte (2001) suggested an existence of an instrument similar to the hieroglyphic of the goddess Seshat whose existence is, however, not confirmed by textual evidence.

4 Horizontal Alignment of Stars

In their text "Astronomical orientation of the pyramids" published as an answer to the paper of K. Spence, D. Rawlins and K. Pickering firstly proposed to use a horizontal alignment of stars to explain the orientation of pyramids. The authors noticed that two stars—Thuban and 10 Draconis—were equidistant from the Pole in 2627 BC and that "when both stars were at the same altitude, north was the direction bisecting them" (Rawlins and Pickering 2001, 699). The proposal is displayed in Fig. 15.

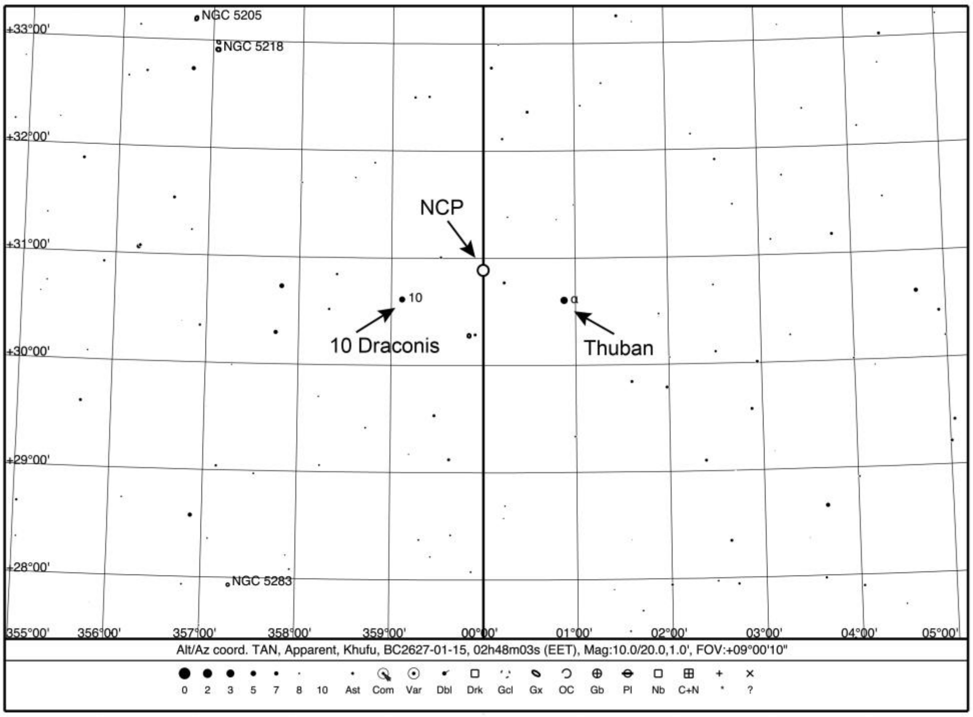

Fig. 15: Horizontal alignment of Thuban and 10 Draconis at the latitude of Khufu's pyramid on January 15, 2627 BC. Map produced on *Cartes du Ciel*.

Although relatively faint (4.5-5.0 mag), 10 Draconis is recorded in all large naked-eye star catalogs but without its hieroglyphic identification and without chronologies supporting the corresponding dating of the pyramids (which implied dates of 2638 BC for Khufu's pyramid), the proposal has not gained much support in Egyptology. The same question can be posed here: on what grounds did the ancient Egyptians conclude that Thuban and 10 Draconis could be used at this particular position to mark the direction towards the Celestial Pole? Nevertheless, the idea that the orientation of the pyramids might have been based upon the stars aligned not only vertically but also horizontally was very fruitful and deserved more attention.

Recently, the text entitled "Stretching of the Cord Ceremony for Astronomical Orientation of the Old Kingdom Pyramids" was released at the internet portal *academia.edu* by Alexander Puchkov (2019).[58] The main idea of the author was that the *primary* target of the astronomical observation was the *Meskhetiu* asterism observed at a special position. According to the inscriptions in the Temple of Hathor at Dendera,

> [The king] stretches the cord in joy, having turned his face **m aqA** Meskhetiu, and establishes the temple of the [Mistress of Dendera] as before.[59]

The same terminology is used in the Temple of Horus at Edfu:

> [The king]…looking at the sky and observing the stars, turns his eyes **m aqA** Meskhetiu.[60]

[58] The reworked text entitled "Multi-star target model for astronomical orientation of the Old Kingdom Egyptian pyramids" is now in press.
[59] Žába 1953, 59, C a, Pl. II; Dümichen 1877, Pl. XLIV. English translation of A. Puchkov (in press, courtesy of the author).
[60] Žába 1953, 59, C b, Pl. II; Chassinat 1928, 167, Pl. LXIV. English translation of A. Puchkov.

This culturally very important constellation is nowadays commonly agreed upon to be the asterism of the Big Dipper in the constellation of the Ursa Major.[61] The technical term *aqA* was translated by R.O. Faulkner (1988) as 'brilliant' or 'blessed' and can thus be interpreted as a 'brilliant star of the Plug'.[62] According to A. Puchkov, this term probably denotes a special sacral position of the asterism as was already proposed by some scholars; the possible interpretations varied from 'culmination' (although the stars of an asterism could not culminate simultaneously) to a not specified position.[63] The ingenious idea of A. Puchkov was to interpret *aqA* as the position of the Big Dipper when two stars—Dubhe and Alkaid—were aligned horizontally;[64] according to the trend in the orientation of the pyramids, the author considered the upper position (see Fig. 16). We do not intend to repeat all the arguments of the author; the interested reader can find them in the original text which provides a detailed analysis of collected information.

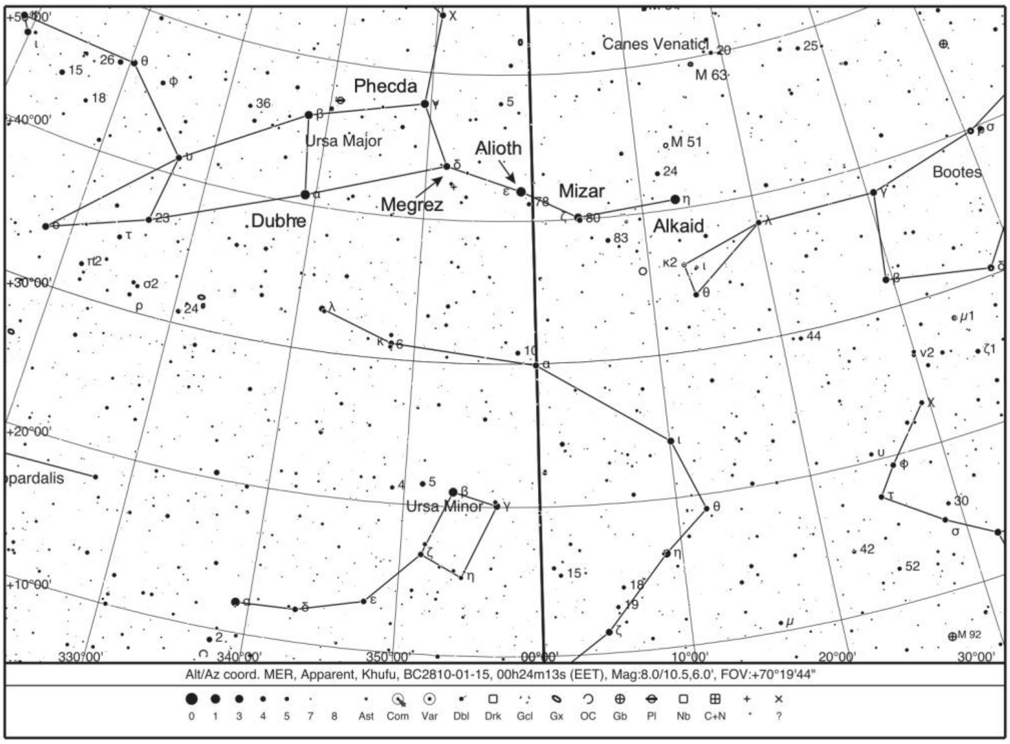

Fig. 16: Horizontal upper alignment of Dubhe and Alkaid at the latitude of Khufu's pyramid on January 15, 2810 BC. Map produced on *Cartes du Ciel*.

The *secondary* target of observation for the pyramids along the main trendline was, according to A. Puchkov, the star Thuban which was observed at the moment of the sacral position of *Meskhetiu*, and the direction towards this star was used to align the sides of the pyramids: "They could consider the 'motionless' central star (the pole star) as the only place that allows the soul of the deceased king to safely move from our 'non-rotating' world to the

[61] Renouf 1874; Neugebauer and Parker 1969, 183; Parker 1974, 60. Berio (2014, 44) identifies it as Boötes.
[62] Belmonte 2001, S7; Shaltout and Belmonte 2005, 290–291; Miranda et al. 2008, 57.
[63] As the American astronomer E. C. Krupp formulated, "The texts mention the *ak* of the Big Dipper, but we don't know what *ak* means. Most likely it refers to a particular position and orientation of the Plough in its circular course around the Pole"(Krupp 1983, 26).
[64] Such a configuration happens twice in 24 hours but, of course, only one of the alignments is observable.

rotating firmament and be adopted in the sky among the stars" (Puchkov 2019, 25).[65] Thus determined, the trendline of the azimuth of Thuban was compared by the author with the trendline of the azimuths of the pyramids to link the absolute and relative chronologies; the method led to 2810 BC as starting date for the Stretching of the Cord Ceremony for Khufu's pyramid. To justify such a change of chronology, the author referred to radiocarbon dating of the Egyptian samples from the Early Dynastic period to the Middle Kingdom (Bonani et al. 2001). The results pointed towards a discrepancy for the structures from the 3th to the 8th Dynasties of about two centuries to the past but become in good agreement for the two pyramids of the 12th Dynasty.[66] A. Puchkov also refers to the chronology proposed by J. Breasted (1906–1907, I, 40–47) who suggested dates about 280 years earlier for the 4th Dynasty. The starting dates for the alignment ceremonies used by A. Puchkov (col. 3) for the synchronization of the relative and absolute (astronomical) chronologies together with his new proposed dates (col. 4) are given in Table 3; the author gives ± 5 years as a confidence interval.

Ruler/pyramid	Dynasty	Date of alignment	Date of alignment (Puchkov)
Djoser	3	2666	2888
Snofru-Meidum	4	2636	2858
Snofru-Bent	4	2619	2841
Khufu	4	2588	2810
Djedefre	4	2565	2787
Khafre	4	2557	?(2779)
Menkaure	4	2531	2753

Table 3: Dates of alignments of ancient Egyptian pyramids according to A. Puchkov.

The azimuths of the pyramids (line 'c') vs. azimuth of Thuban (line 'a') for the dates of alignment proposed by A. Puchkov are plotted in Fig. 17. Obviously, the gradients of the two lines differ and the graph reveals a secular trend implying decreasing precision of the alignments that is difficult to explain.

[65] Note that this justification is only valid for the new chronology proposed by the author.

[66] To the contrary, the study of Ramsey et al. (2010) based upon short-lived samples taken from the museums is generally consistent with the conventional chronologies. This study estimates the accession date for the reign of Djoser as between 2691 and 2625 BC in comparison with 2667 BC by J. Malek (Shaw 2000, 482) and 2633 BC by von Beckerath. It is these discrepancies between the results of radiocarbon dating, caused—apart from the limited number of samples—by non-continuous series of dendrological data and by barely considering the 'old wood' and the 'old water' reservoir problems which lead Egyptologists to generally refrain from accepting the earlier dating of the Old Kingdom monuments. As S. W. Manning (2006, 340) stated: "the radiocarbon 'dates' thus can be valid/correct—*but* they date the 'old' wood (etc.) and not the cultural/historical target date wanted: the building of the pyramid monument." Another problem was addressed by Manning (2006, 338) as follows: "A key issue is the history of past natural radiocarbon levels; there was in effect a plateau in radiocarbon levels in the period 2900–2500 BC. This means that radiocarbon ages for the period 2900-2500 BC typically could intercept at several places with the radiocarbon calibration curve (i.e. several calendar periods have similar radiocarbon ages)".

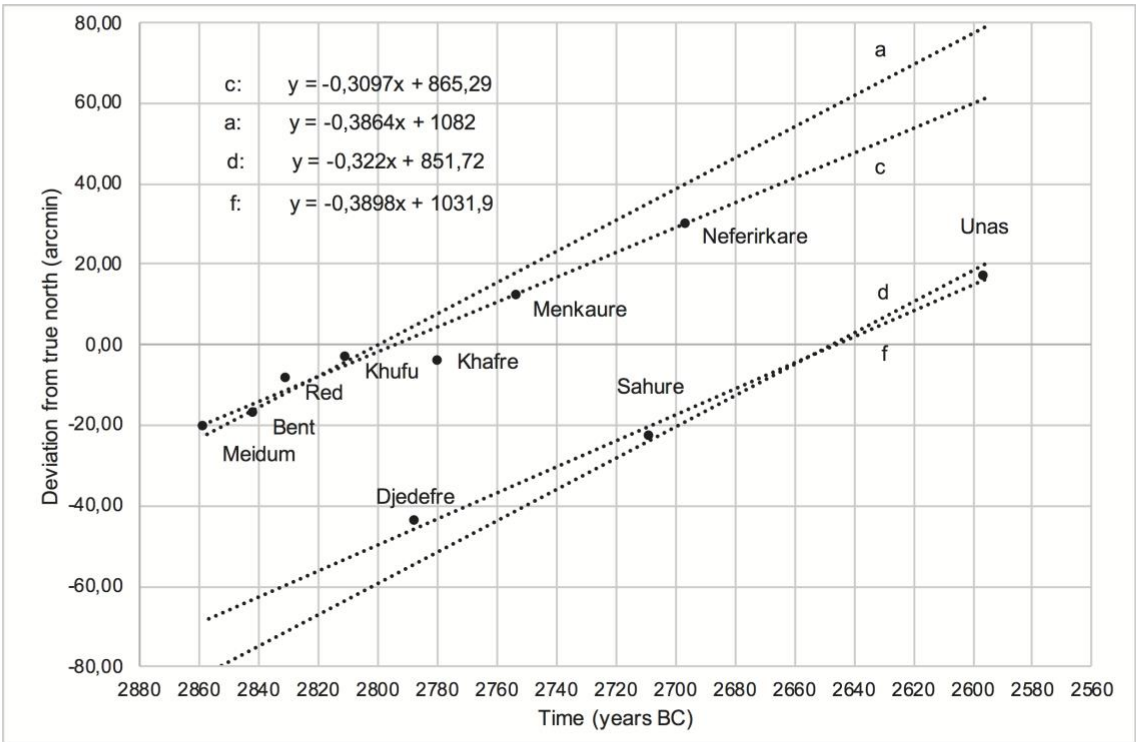

Fig. 17: Deviations of pyramid orientations (east sides) from true north over time vs. azimuths of Thuban and Alioth at the moment of the upper horizontal alignment of Dubhe and Alkaid (Puchkov's new chronology). The dashed line 'a' is a trendline through the points corresponding to the azimuth of Thuban and the dashed line 'f' is a trendline through the points corresponding to the azimuth of Alioth.

The deviations in the orientation of the pyramids towards Thuban at the moment of the horizontal alignment of Dubhe and Alkaid calculated as dA – DA (where dA are the azimuths of the east sides of the pyramids and DA are the azimuths of Thuban) for the corresponding place and assumed time of the foundation ceremony are given in Fig. 18. For the new proposed chronology, the mean deviation of the azimuths of the pyramids along the main trendline 'c'[67] from the position of Thuban is reduced to ca. 2´ and the standard deviation from the mean value is ca. 5´.[68] Clearly, the precision of the orientation of the pyramids towards Thuban degrades over time and this is not the case for K. Spence's 'synchronization' (compare Figs. 6 and 18).

[67] Here, the trendline 'c' is drawn once more without considering the data of Khafre's pyramid.

[68] For comparison, in the new chronology proposed by Spence (2000), the mean deviation attains 2.04´ and the standard deviation is about 2.4´.

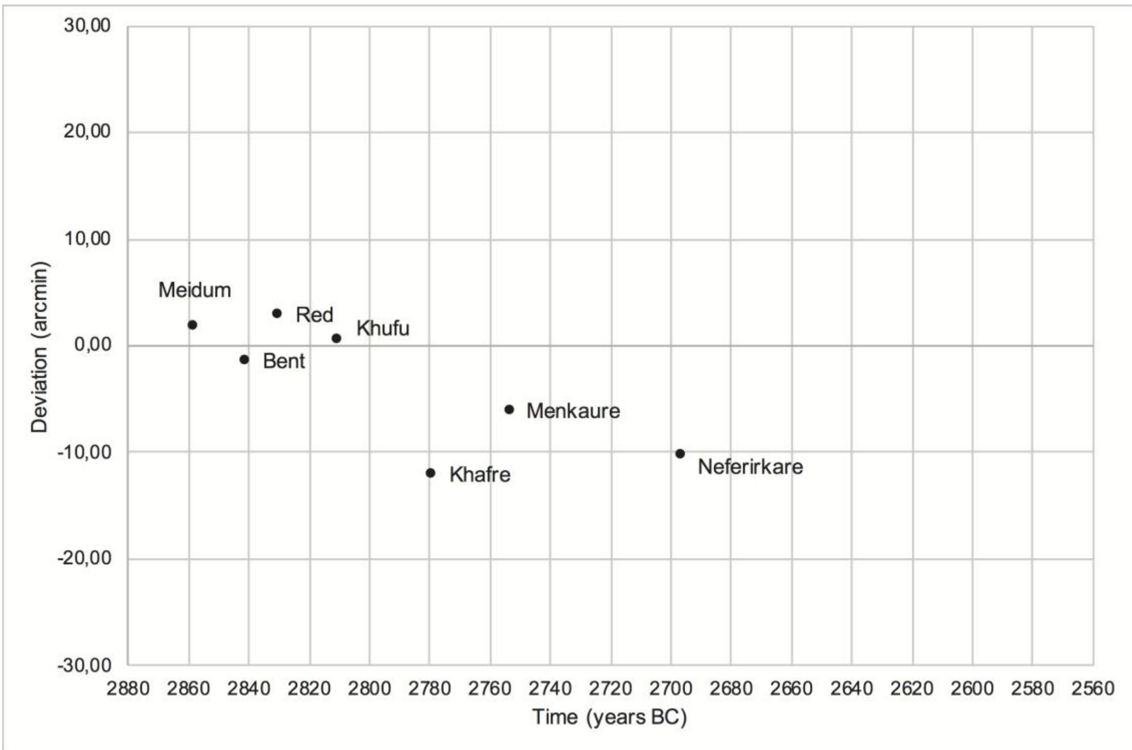

Fig. 18: Precision of pyramid orientations (east sides) towards Thuban at the moment of the upper horizontal alignment of Dubhe and Alkaid (Puchkov's new chronology).

A potential additional argument for the validity of the method could be the number of *other* pyramids whose orientation can be linked with the moment of the horizontal alignment of Dubhe and Alkaid. A. Puchkov explains the second trendline 'd' as a result of targeting the star Alioth at that peculiar moment (line 'f' at Fig. 17);[69] the mean value of the deviation of this orientation is 2.9´ and the standard deviation from the mean value is about 7.0´. In addition, the author suggests that Sekhemkhet's and Khaba's pyramids (both badly preserved) were oriented towards Phecda and Megrez, respectively, at the moments of the horizontal alignment of Dubhe and Alkaid. Involving three additional stars in the explanation, however, poses a question: provided that the horizontal alignment of Dubhe and Alkaid worked as a time marker for targeting different stars, why was this target for Snofru, Khufu, Menkaure and Neferirkare associated with Thuban, i.e. with a star close to the North Celestial Pole (for the date proposed by the author) while for other rulers these targets were Alioth, Phecda, and Megrez, which are not closely connected with the concept of the centre of the apparent celestial rotation? A. Puchkov suggests that a normal rule of alignment was "one pyramid for one star of the *Meskhetiu"* and that Snofru was the first to digress from this rule (private communication).

We assume that—provided that radiocarbon dating gains more acceptance between Egyptologists and a shift of about two centuries to the past will be agreed upon—the method may be considered as a viable solution. The latest publication of Gautschy et al. (2017), based upon the new data from the Old Kingdom for the heliacal rising of Sirius, supports, however, the classical chronological models and gives e.g. for Khufu's accession date the range of 2503–2636 BC.

[69] The dates of the alignments of these pyramids, although not given directly in Puchkov's text, can be easily calculated according to his new chronology.

The alignment of Dubhe and Alkaid is not the only possible horizontal alignment in the constellation *Meskhetiu*. We will show now that two other prominent bright stars—Alioth and Mizar—aligned horizontally tend to support von Beckerath's chronology with Mizar being a target of observations at the moment of such horizontal alignment. Provided that the term *aqA* denotes a special position of stars in the *Meskhetiu*, this position could as well be a horizontal alignment of Alioth and Mizar. Fig. 19 shows such alignment at the latitude of Giza in 2552 BC and Fig. 20 displays the deviations of the azimuth of the east sides of the pyramids from true north over time vs. the azimuth of Mizar at the moment of the horizontal alignments of these two stars. According to our calculations, the position of Mizar at the moment of its horizontal alignment with Alioth marked the meridian of Giza in 2548 BC.

The secondary trendline can be explained in this case either as the orientation towards Kochab (the trendline 'f' corresponds to the azimuth of Kochab at the moment of the horizontal alignment of Alioth and Mizar) or to ζ UMi (trendline 'g').

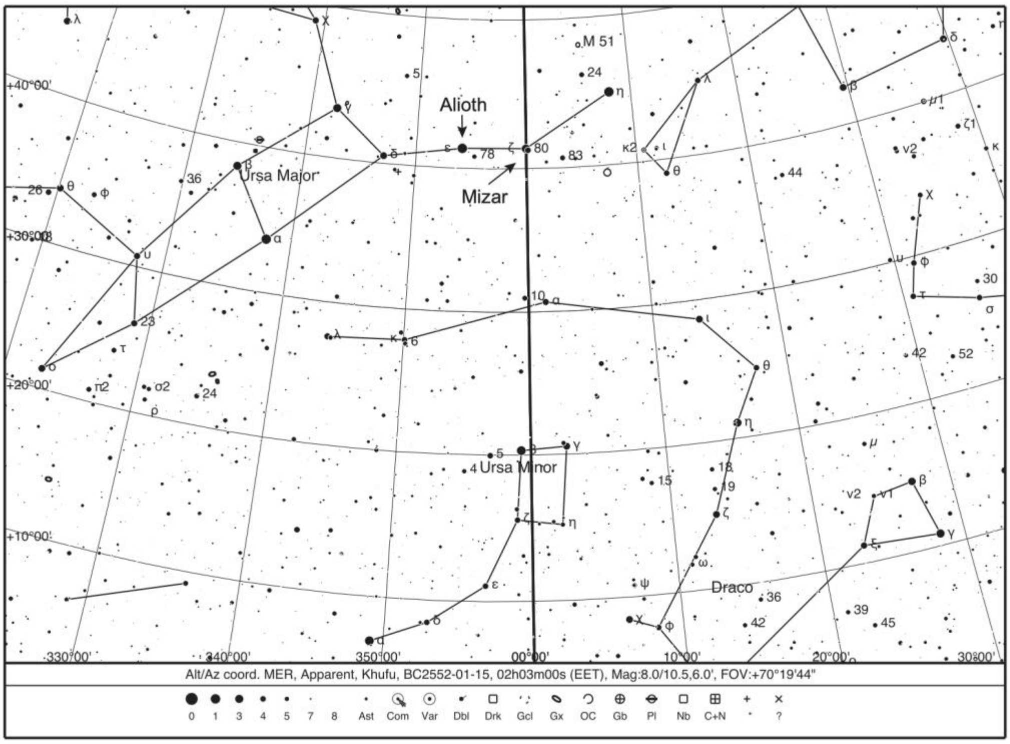

Fig. 19: Horizontal upper alignment of Alioth and Mizar at the latitude of Khufu's pyramid on January 15, 2552 BC. Map produced on *Cartes du Ciel*.

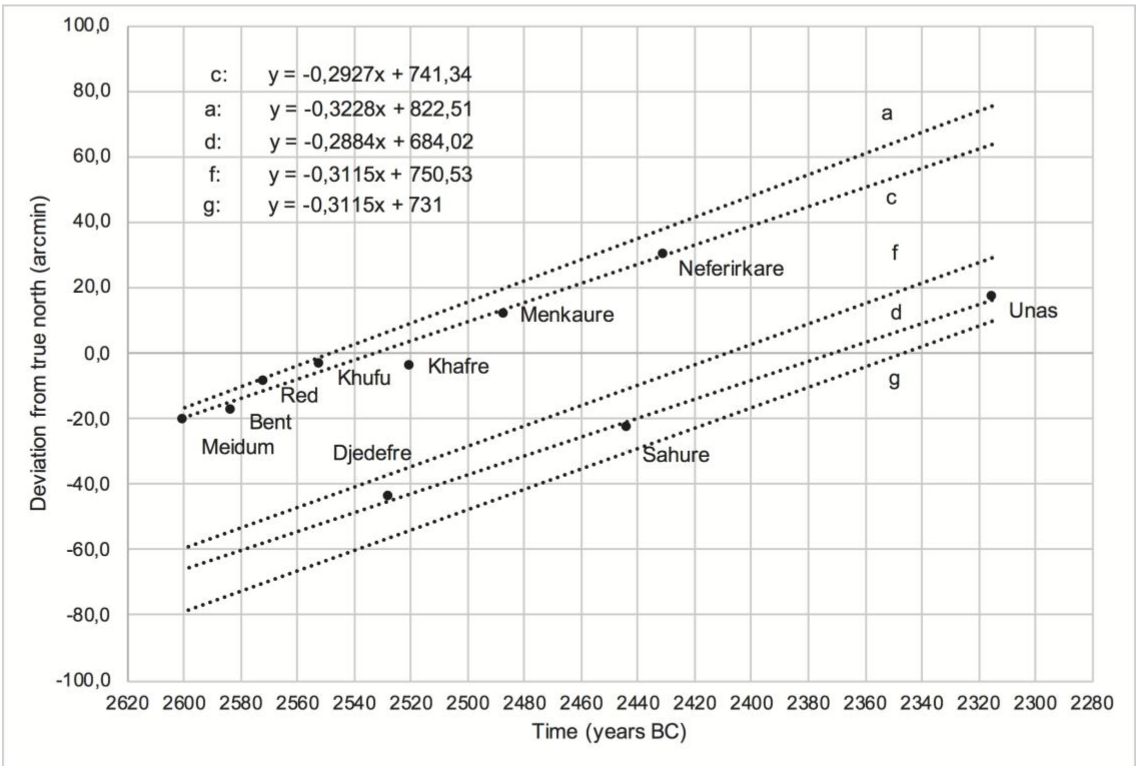

Fig. 20: Deviations of pyramid orientations (east sides) from true north over time vs. azimuths of Mizar, Kochab, and ζ UMi at the moment of the upper horizontal alignments of Mizar and Alioth (von Beckerath's chronology.) The dashed line 'a' is a trendline through the points corresponding to the azimuth of Mizar, the line 'f 'is a trendline trough the azimuths of Kochab, and the line 'g' is a trendline through the points corresponding to the azimuth of ζ UMi.

The errors of the orientation of Snofru's, Khufu's, Menkaure's, and Neferirkare's pyramids towards Mizar at the moment of the horizontal alignments of Mizar and Alioth, calculated as dA – DA are displayed in Fig. 21. In comparison to the method proposed by J. A. Belmonte (Fig. 10 and 12), these errors reveal no secular trend over time and stay at a lower level. For von Beckerath's chronology, a mean deviation of the orientation of the pyramids towards the stellar vertical is -4.7´ with a standard deviation of 3´. Comparison of Figs. 17–18 and 20–21 shows that there exists a solution based upon horizontal alignment of stars in the *Meskhetiu* which does not demand any striking modification of the existing chronologies and matches the main trendline in the azimuths of the pyramids better than method based upon the horizontal alignment of Dubhe and Alkaid.

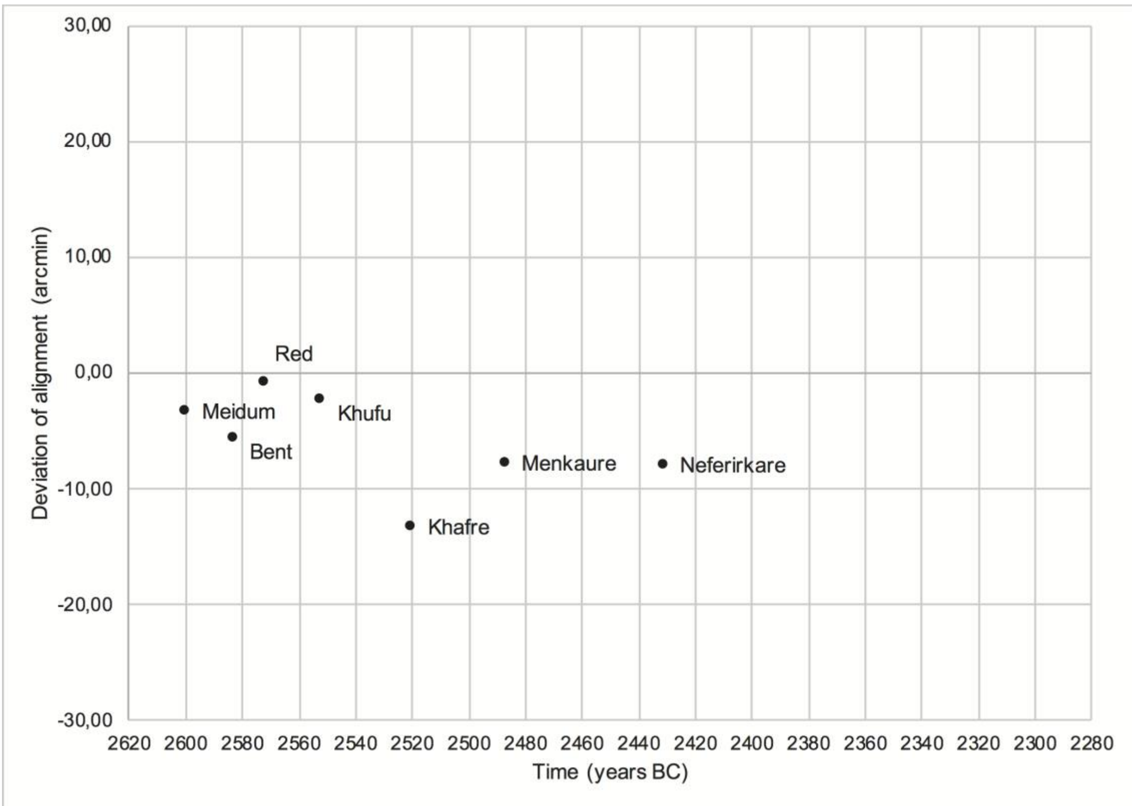

Fig. 21: Precision of pyramid orientations (east sides) towards the upper horizontal alignments of Mizar and Alioth (von Beckerath's chronology).

However, both methods have a problem of observation of the corresponding stellar configuration at high altitudes. A. Puchkov (2019, 6–9) assumes that the poles and the looped cord were intended to create an artificial horizon and that both stars were observed when they simultaneously appeared between two parallel threads of the cord. The required precision of the method is, however, of the order of some arcminutes and could not be achieved without horizontal levelling. Because this levelling must have been realized on a surface high enough to cover the corresponding segment of the sky, we assume that the horizontal alignment of stars was somewhat problematic to use for the orientation of the pyramids.[70] Even if a particular stellar alignment had a sacral meaning, a simpler target close in time to this event might have been used by ancient architects.

5 A New Proposal

Both Alioth and Mizar as well as the pair of stars proposed by J. Belmonte belong to the constellation *Meskhetiu* mentioned in the funeral ceremonies. The *Meskhetiu* was often represented in textual and pictorial evidence as a bull's foreleg, a stylised ovoid bull body (the tomb of Senenmut at Deir el-Bahari, ca. 1450 BC) or as a more realistic bull image (the burial chamber of Seti I in the King's Valley, ca. 1290 BC). Although initially based upon observations, the outlines of the constellations in astronomical diagrams represent the scribes

[70] In Puchkov's new chronology, Thuban is close to the North Celestial Pole and a measurement error in the position of the horizontal alignment of Dubhe and Alkaid would not greatly influence the azimuth of Thuban, but it would noticeably change the azimuths of Alioth, Phecda and Megrez.

and artists' work and cannot be considered without their mythological and religious background. Another important asterism mentioned in the Pyramid Texts is the *Mooring Post*:

> The sky's door has been opened to you, the Cool Waters' door that bars the subjects has been pulled open to you. The Mooring-Post will care for you, the populace will call out to you. the Imperishable Stars will wait for you.[71]

Many ritual spells mention the asterisms *Two Adzes*[72] which played an important role in the *Opening of the Mouth* ceremonies[73] and would have helped to 'hack' the sky on the way to the Imperishable Stars. It is accepted that the *Adzes* refer to the asterisms of similar form, in the Big and Little Dipper.[74] According to A. Roth, the constellation of the Big Dipper was compared to both the adze and to the foreleg of a bull: "Both the foreleg and the adze were added to the offering ritual at the same time, in the Pyramid Texts of Mernere, and their association there was probably due to their common association with this constellation. This stellar element was presumably connected principally to the realm of the dead, which in some conceptions of the afterlife was clearly located in the region of the circumpolar stars. The orientation of the stellar adze/foreleg towards the circumpolar stars is similar to the orientation of the mouth-opening implements towards the mummy/statue in depictions of the ritual. Even before the adze and foreleg were introduced into the offering ritual of the Pyramid Texts, its text was placed on the north wall of the burial chamber of Unas, hinting at the same relationship with the northern circumpolar stars"(Roth 1993, 70–71). That the same asterism could have been addressed in different ways was not unusual: "This type of duplicate symbolism, fusing different elements that have similar effects, is extremely common in Egyptian rituals" (Roth 1993, 76).

In typical representations, the constellation *Meskhetiu* was not displayed with any special *vertical* or *horizontal* arrangement of stars (Figs. 22–24), and the linear arrangement of three stars in the tail of the bull in Senenmut's astronomical diagram is definitely inclined; thus J. Belmonte's and A. Puchkov's proposals are not supported by pictorial evidence.[75] The asterism *Mooring Post*, was usually represented as two daggers (often one in the form of a small crocodile, and occasionally only one dagger is displayed) in the hands of the most remarkable stellar constellation in the astronomical diagrams, the *Female Hippopotamus*. These daggers are depicted as parallel *vertical* objects, variously interpreted in modern literature. K. Locher located them in the tail of the Draco and in the Little Dipper;[76] J. A. Belmonte seems to be in agreement (at least, partly) with this suggestion.[77] In some iconographic traditions, a chain connects one of the daggers with the tail of the Bull: In the Ramesside *Book of Day and Night* it is stated that "as to this Foreleg of Seth […], it is in the northern sky, tied to two Mooring posts of flint by a chain of gold. It is entrusted to Isis as Hippopotamus (*rrt*) to guard it."[78] The

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[71] The Pyramid Texts of Pepi I Spell 313 (Allen 2005, 122).
[72] Unis Spell 165 (Allen 2005, 46), Teti I Spell 194 (Allen 2005, 79), Merenre Spell 399 (Allen 2005, 235), Pepi II Spell 100 (Allen 2005, 252), Pepi II Spell 296 (Allen 2005, 260).
[73] Roth (1993: 57–79). The earliest occurrence of the ritual is probably a text from the valley temple of Snefru at Dahshur (Otto 1960, II, 3.)
[74] Krupp 1983, 211-213; Belmonte 2001, S6; Lull and Belmonte 2009, 188. For duplicate representation of the Egyptian constellations see Lull and Belmonte 2009, 187–189.
[75] The later ceiling detail in the tomb of Pediamenopet (26th dynasty) shows a couple of stars between the targeting triangle and one of the *Mooring Posts* which—if not an artist's interpretation—could indicate a change in the method of stellar orientation over time (Fig. 25).
[76] Locher (1985) considered the first of the posts to be built up by γ Umi and 5 Umi at the bottom and α Dra at the top, and the second post to have been β Umi and 4 Umi at the bottom and κ Dra at the top assuming that the tops of the posts pointed towards the position of the NCP in the third and in the second millenium BC, respectively. See also Locher 1990, S49–51. This interpretation does not consider a visual vertical alignment of the sides of the Little Dipper.
[77] Lull and Belmonte 2006, 9; Lull and Belmonte 2009, 163, Table 6.2. 9.
[78] Piankoff and Drioton 1942, 24, 95 (in translation of Wilkinson 1991, 151).

Ptolemaic *Papyrus Jumilhac* also states that “the Great Hippopotamus (*rrt wrt*) holds it so that it cannot travel among the gods.”[79] In Seti I’s astronomical diagram (Fig. 24) a male figure *uses this chain to climb to the Meskhetiu*. Of course, “the utilization of astronomical paintings must proceed with great caution”(Wilkinson 1991, 149) and symbolically they could be envisaged in a number of ways, but identification of the tops of two *Mooring Posts* with the North Celestial Pole at different times (as K. Locher proposed) is, in our opinion, not plausible. Firstly, we have no evidence for the existence of precise stellar maps which would allow ancient people to notice and later display the change in the stellar positions relative to true north due to precession.[80] Secondly, both posts are represented as attached to the ground and thus do not permit representation (even symbolically) of any *circumpolar* motion. Additionally, the texts do not say that the *Meskhetiu rotates* about one of the *Mooring Posts*: they just say that both constellations are *connected* by a chain which holds them tied together and constrain their motion.

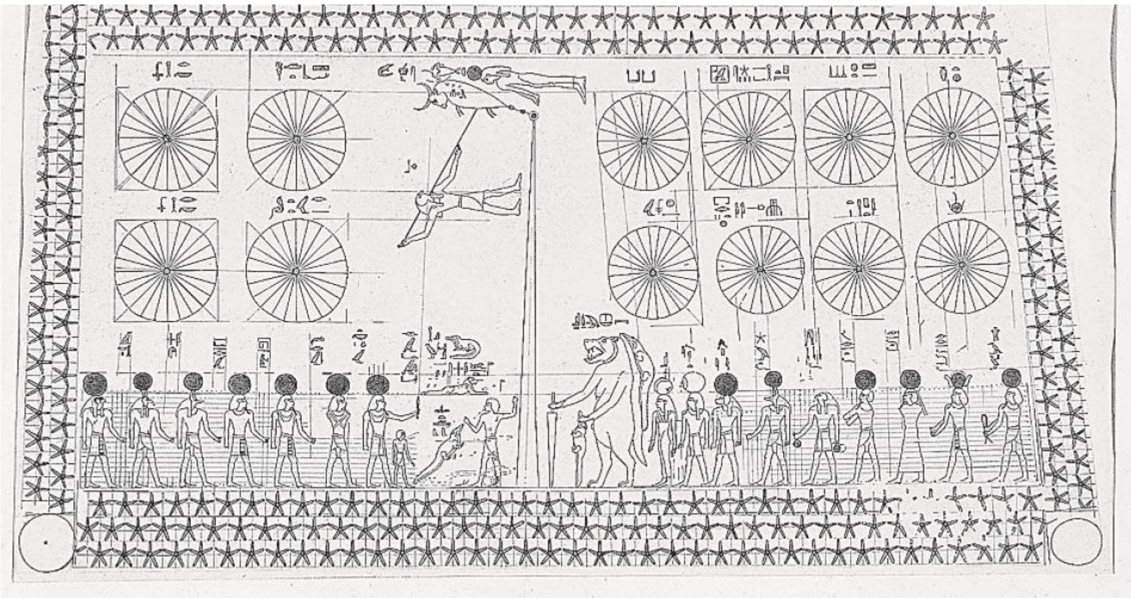

Fig. 22: Ceiling detail. Northern panel of Senenmut’s astronomical diagram. After C. Wilkinson (The Met’s Open Access).

[79] Vandier 1961, 108, 129 (in translation of Wilkinson 1991, 151).

[80] Identification of one of the tops of the *Mooring Posts* with the pole of ecliptic mentioned by Lull and Belmonte (2006, 12, Table 2), whose position could not be observed in the night sky, seems even more anachronistic because it presumes exact knowledge of the position of the big circle of the ecliptic and the mathematical concept of the corresponding pole.

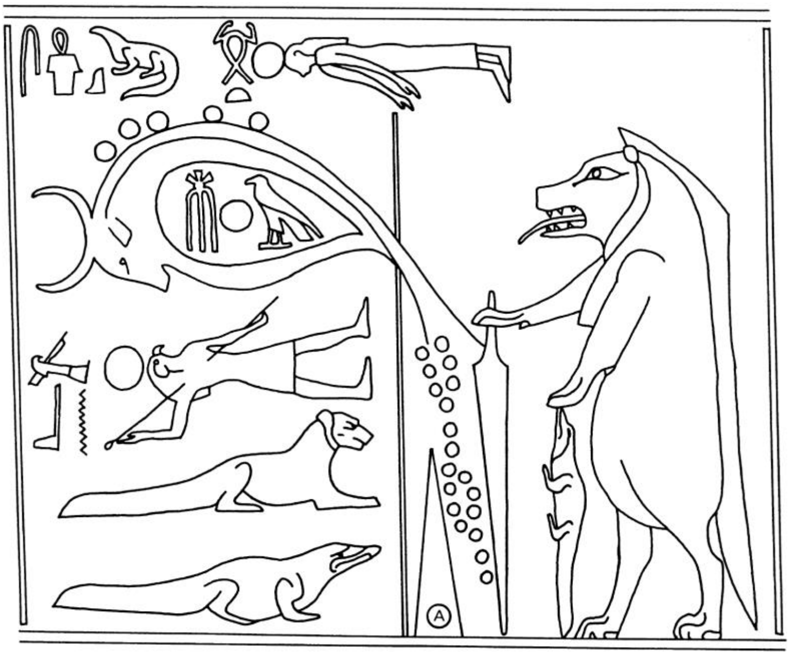

Fig. 23: Ceiling detail, Ramesseum, Second Hypostyle Hall, Temple of Ramses II. After Wilkinson 1991, Fig. 3.[81]

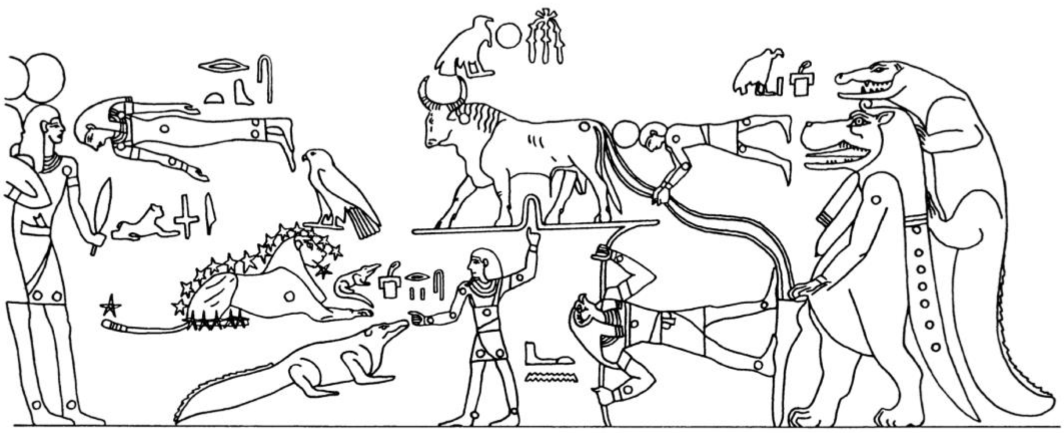

Fig. 24: Ceiling detail, tomb of Seti I, Sarcophagus Chamber C. After Wilkinson 1991, Fig. 2.

[81] Figs. 23–25 are given by R. H. Wilkinson with the reference to Neugebauer and Parker (1969) and reproduced by permission of the American Research Center in Egypt, Inc. (ARCE).

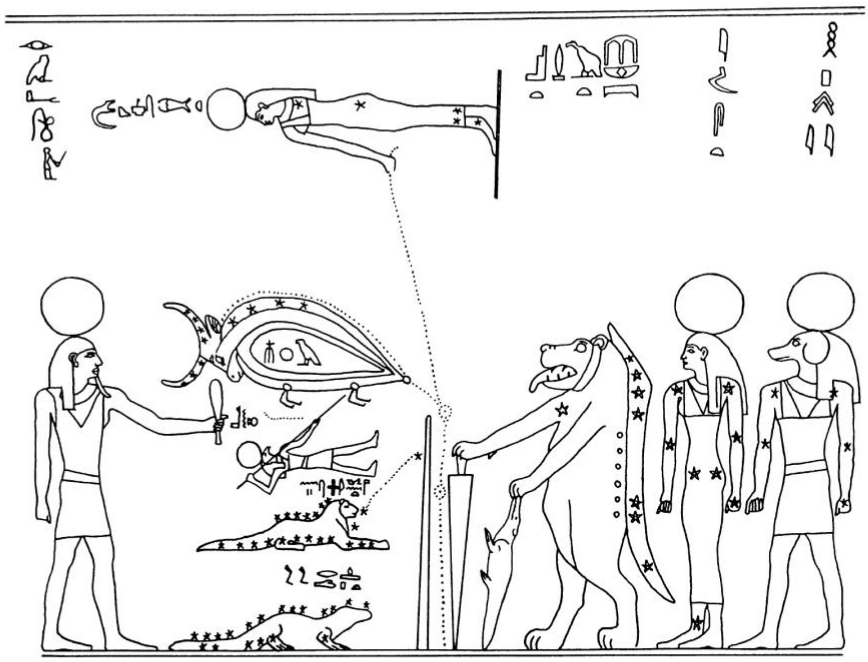

Fig. 25: Ceiling detail, tomb of Pediamenopet (26th dynasty). After Wilkinson 1991, Fig. 4.

We propose here the interpretation of the *Mooring Posts* as two sides of the Little Dipper represented by the pair Kochab and ζ UMi, and by the pair γ and η UMi at the moment when they were aligned vertically as shown in Fig. 26 for the latitude of Khufu in 2552 BC. It is exactly this position which corresponds visually to the upper position of the Bull's Foreleg as displayed in the existing astronomical diagrams.

The passages from the *Book of Day and Night* and the *Papyrus Jumilhac* link the apparent motion of the the Big and Little Dipper in a natural way: both asterisms could have been observed simultaneously, visually rotating in the course of the night as though chained together, giving a strong impression of a connection between the two asterisms. We assume also that the vertical alignment of Kochab and ζ UMi might have been considered to be a kind of a stairway to the king's final destination:[82]

> A stairway to the sky is set for you among the C. S. [Circumpolar stars /Imperishable Stars].

As seen in Fig. 26, the position of the North Celestial Pole lay approximately in the middle of the line connecting the stars Mizar and Kochab;[83] it is a small difference in the

[82] Faulkner 1966, 156, Nt. 773–74. In the Pyramid Texts of Pepi I, the king is addressed as "you of the mooring, who came from the *Mooring Post*" (Allen 2005, 102, Pepi I Spell 14). See also Merenre Spell 399 (Allen 2005, 235).

[83] The later Dendera Zodiac dating to the reign of Ptolemy XII and his daughter Cleopatra VII mixed the symbolic presentations of the ancient Egyptian, Mesopotamian, and even Greek constellations. The *Meskhetiu* and the *Female Hippopotamus* (whose position is rotated by 90° and is mirror inverted relative to the bull's foreleg exactly as in the astronomical ceiling of the Late Period tomb of Petosiris) are displayed in the center part of the diagram. Although the relief is not to scale and has a symbolic rather than an astronomical meaning, one cannot claim—as Lull and Belmonte (2009, 188) did—that the diagram is centered around the top of the adze in the hand of the *Hippopotamus,* which can therefore be identified as the Celestial Pole. In the proportions of the relief, the position of the Pole instead lies on the chain connecting the top of the *Mooring Post* with the Bull's foreleg (where a small figure of a Jackal is also depicted) which is fully in agreement with our interpretation.

alignments of stars as well as the principal difference of our understanding of the target of pyramid alignments which makes our proposal below different from that of Spence (2000). Although our interpretation of the *Mooring Post* is open to question, the following calculations show that the vertical alignment of Kochab and ζ UMi was, with a high probability, used in the orientation of the pyramids along the main trendline.

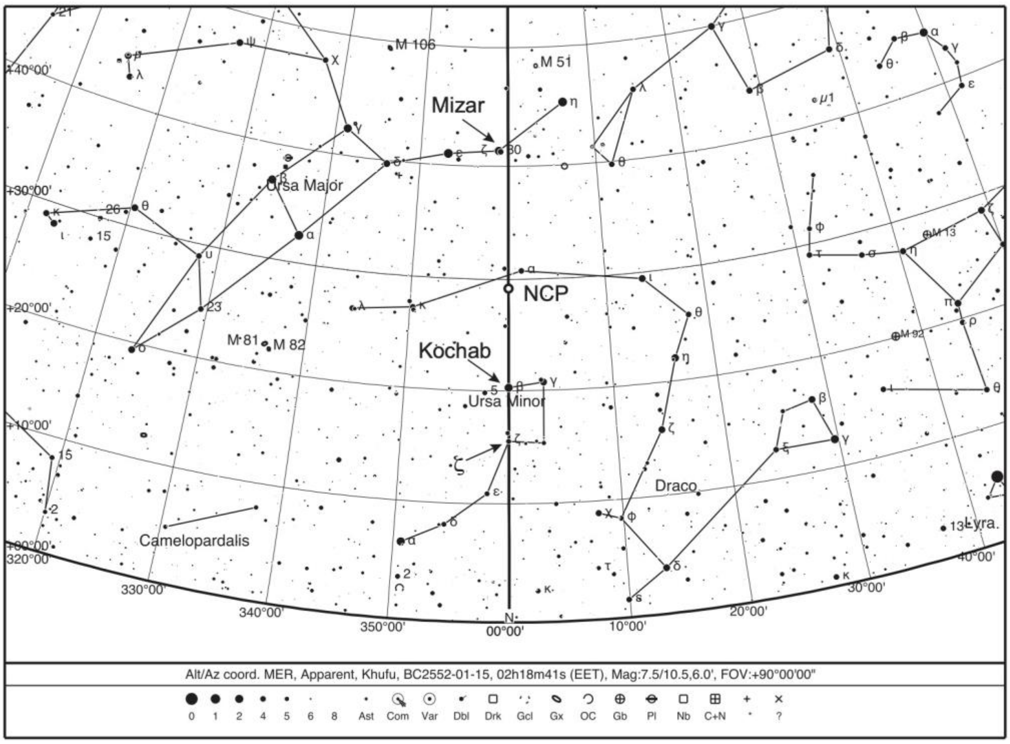

Fig. 26: Vertical lower alignment of β UMi (Kochab) and ζ UMi at the latitude of Khufu's pyramid on January 15, 2552 BC. Map produced on *Cartes du Ciel*.

We start with the chronology discussed in Hornung et al. (2006) and display the azimuths of the vertical alignment of Kochab and ζ UMi against the azimuths of the east sides of the pyramids in Fig. 27; the results make it evident that this chronology does not support our proposal.

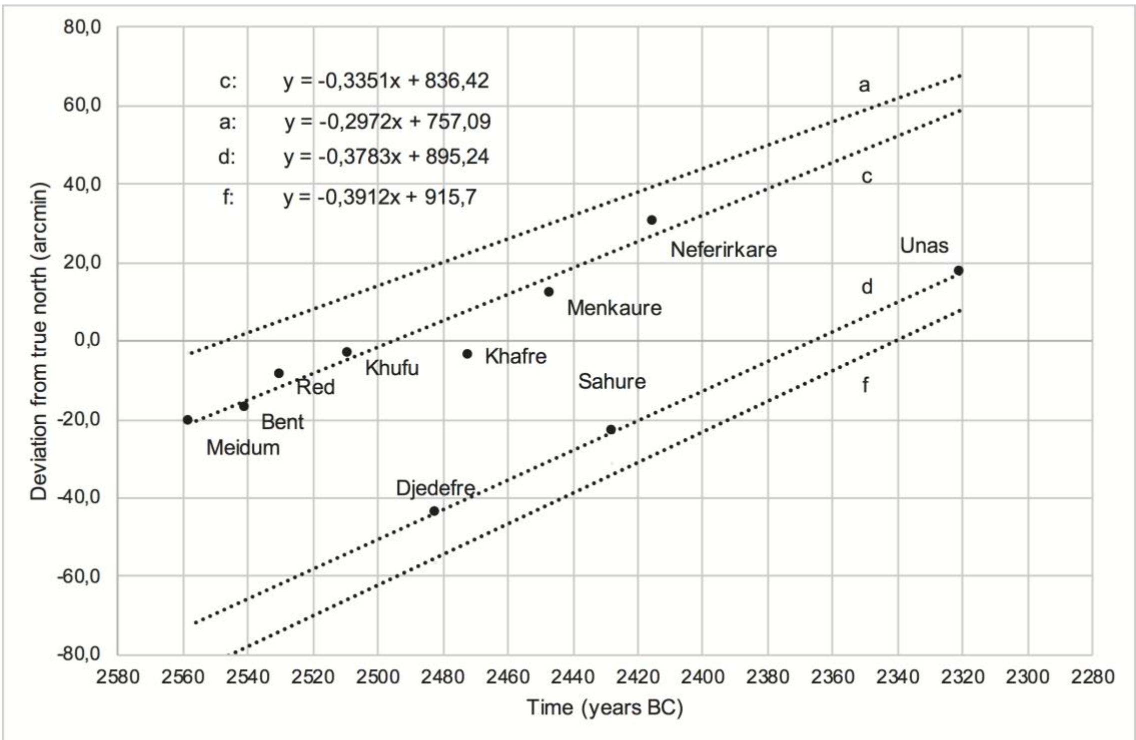

Fig. 27: Deviation of pyramid orientations (east sides) from true north over time vs. azimuth of the lower vertical alignment of Kochab and ζ UMi (Hornung's chronology). The dashed line 'a' is a trendline through the azimuths of this vertical and the dashed line 'f' is the trendline through the azimuths of Mizar at the moments of this vertical alignment.

The next step was to check our new proposal against Shaw's chronology; the results given in Fig. 28 show that the gradient of the trendline 'a' through the azimuths of the vertical stellar alignment is now very similar to the gradient of the main trendline 'c': -0.3001 vs. -0.3073, respectively.

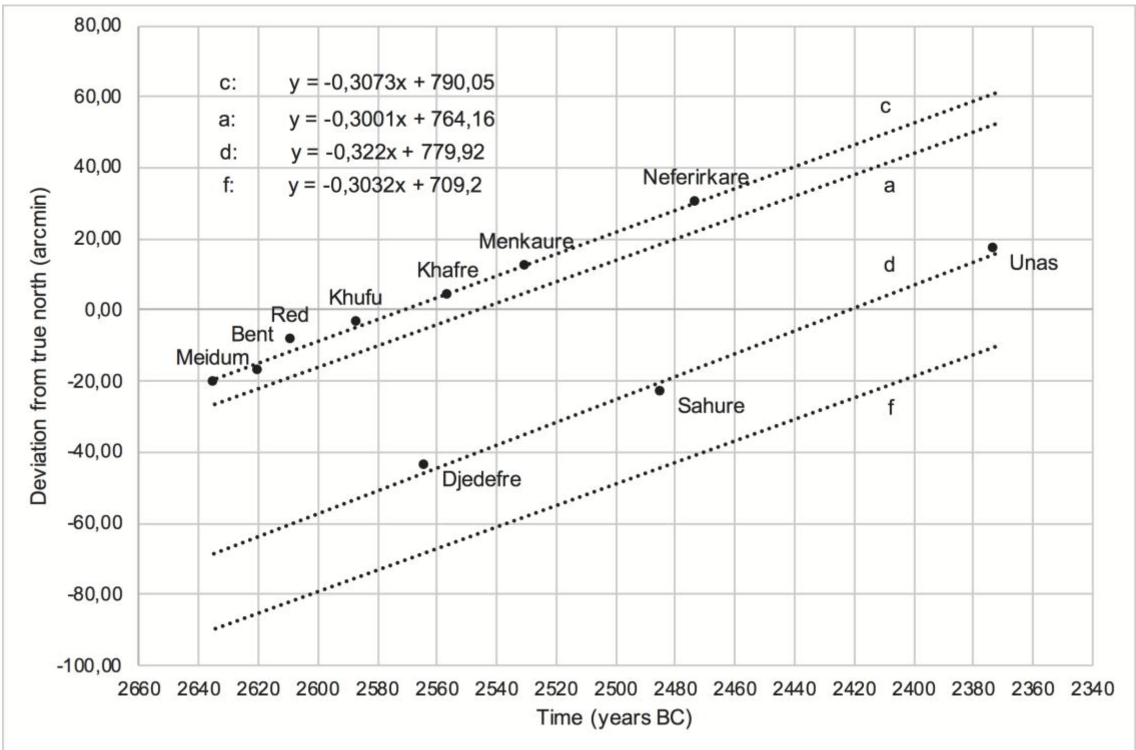

Fig. 28: Deviation of pyramid orientations (east sides) from true north over time vs. azimuth of the lower vertical alignment of Kochab and ζ UMi (Shaw's chronology). The dashed line 'a' is a trendline through the azimuths of this vertical, and the dashed line 'f' is the trendline through the azimuths of Mizar at the moments of this vertical alignment.

In greater detail, the precision of the orientation of the pyramids towards the vertical through Kochab and ζ UMi is given for this chronology in Table 4. Here, the azimuths of the east side of the pyramids are dA, the azimuths of the vertical are DA, and the errors of the orientation, calculated as dA – DA, are given in arcminutes in col. 5.[84] The mean deviation for the pyramids along the trendline 'c' (without Khafre) from the stellar vertical can be estimated as 7.4´ with a standard deviation of 1.9´.

[84] Although the results are given in the table rounded to one arcminute, all the calculations were performed to 15 digits. Once more, the errors of each orientation are calculated not relative to the trendline 'a' but for every particular location and moment of time separately.

Ruler/pyramid	Date of alignment ceremony	dA (east side)	DA of the vertical Kochab and ζ Umi	dA - DA
Snofru-Meidum	2635	-20,6	-27	6
Snofru-Bent	2620	-17,3	-22	5
Snofru-Red	2609	-8,7	-19	10
Khufu	2587	-3,4	-12	9
Djedefre	2564	-43,9	-5	-39
Khafre	2556	-4,0	-3	-1
Menkaure	2530	12,0	5	7
Sahure	2485	-23,0	19	-42
Neferirkare	2473	30,0	22	8
Unas	2373	17,1	52	-35

Table 4. Precision of pyramid orientations (east sides, in arcminutes) towards the lower vertical alignment of Kochab and ζ UMi (Shaw's chronology).

Let us consider now the case of the three other pyramids—Djedefre, Sahure und Unas—whose alignment to true north follows a trendline 'd'. Of course, these pyramids could hardly have been oriented towards the vertical alignment of Kochab and ζ UMi, the errors in the orientation were too large. We suppose that exactly at the moments of the vertical alignment of Kochab and ζ UMi, another object has been targeted, another star which was assumed to be the final destination of the celestial ascent. A hint to the identification of this star is given in the astronomical diagram of Senenmut (Fig. 22) where three stars in the bull's tail are displayed in a line with the third star colored in red and encircled by a red ring—and this star seems to be a target of observations with a sort of astronomical instrument represented by a triangle.[85] The best reasonable candidate is, in our opinion, the star Mizar; the trendline 'f' in Fig. 28 is drawn through the azimuths of Mizar at the corresponding points in time. The mean value of the differences between Mizar's azimuth and the azimuth of the pyramid's east side attains, however, about 24´ which seems to be too imprecise in comparison to the alignment to the vertical through Kochab and ζ UMi.

Our next step was to test the precision of the orientation of the pyramids towards the vertical alignment of Kochab and ζ UMi for von Beckerath's chronology; the results are shown in Fig. 29. The gradients of the two lines become even more similar (-0.2976 vs. -0.2927) and the line 'c' matches the azimuths of the vertical alignment of the two stars (given by the line 'a') better than for Shaw's chronology.

[85] The star was earlier identified as Mizar by Pogo (1930). G. Priskin (2019) identifies it as Vega due to an alternative identification of the *Meskhetiu* constellation. Lull and Belmonte (2009, 161) propose this star to have been Alkaid (η UMa), in agreement with Locher (1990) or Leitz (1991, 1995). In this case, however, Alkaid could not be used for the orientation of the pyramids: a target star should have been observed close to the meridian, and with Alkaid at its upper culmination, the Big Dipper takes a position visually different from that given in astronomical diagrams.

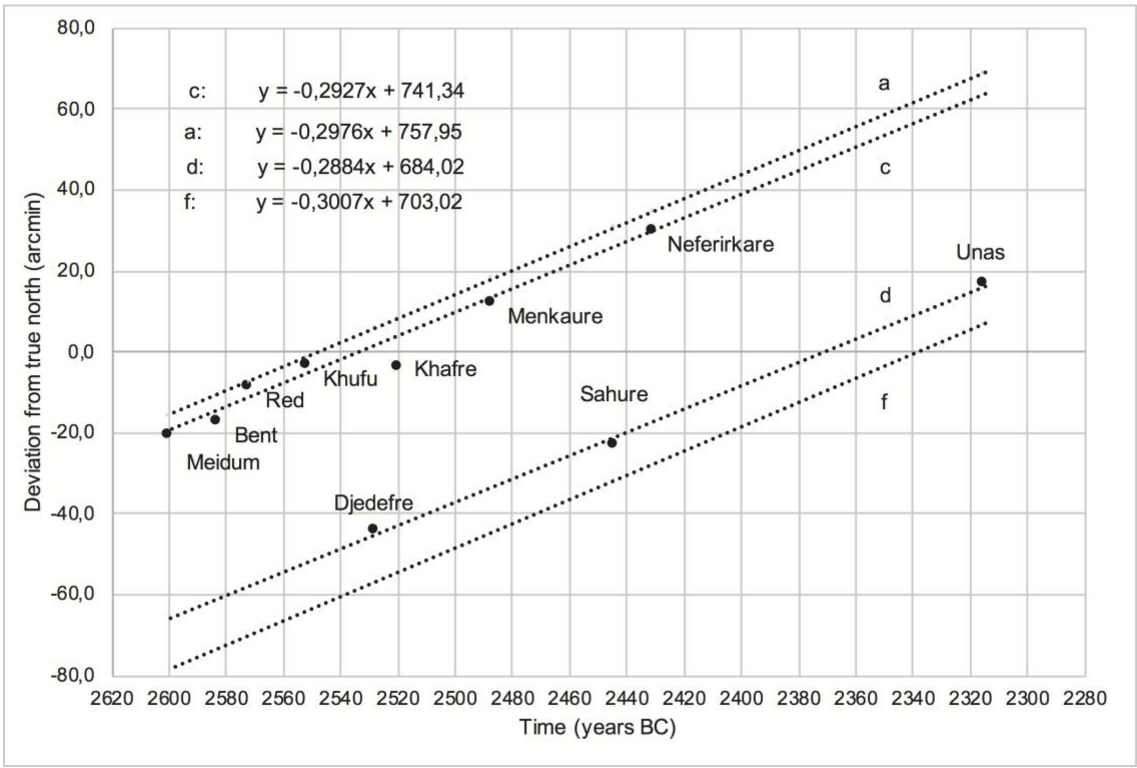

Fig. 29: Deviation of pyramid orientations (east sides) from true north over time vs. azimuth of the lower vertical alignment of Kochab and ζ UMi (von Beckerath's chronology). The dashed line 'a' is a trendline through the azimuths of this vertical, and the dashed line 'f' is the trendline through the azimuths of Mizar at the time of this vertical alignment.

The precision of this orientation calculated for every pyramid separately is given (in arcminutes) in Table 5. For this chronology, the mean deviation of the orientation of the pyramids towards the stellar vertical is -4.1´ with a standard deviation of 2.1´. Such a small mean deviation—close to the limit of naked-eye observations—can be explained as an unavoidable but very small systematic error of the method.[86]

Ruler/pyramid	Date of alignment ceremony	dA (east side)	DA of the vertical Kochab and ζ Umi	dA - DA
Snofru-Meidum	2600	-20,6	-16	-5
Snofru-Bent	2583	-17,3	-11	-6
Snofru-Red	2572	-8,7	-8	-1
Khufu	2552	-3,4	-1	-2
Djedefre	2528	-43,9	5	-49
Khafre	2520	-4,0	8	-12
Menkaure	2487	12,4	18	-6
Sahure	2444	-23,0	31	-54
Neferirkare	2431	30,0	35	-5
Unas	2315	17,1	69	-52

Table 5. Precision of pyramid orientations (east sides, in arcminutes) towards the lower vertical alignment of Kochab and ζ UMi (von Beckerath's chronology).

[86] If we assume zero systematic error (as previous scholars did) and interpret this deviation as a temporal shift, it would correspond to a shift of von Beckerath's chronology by 14 years into the past.

The same results (i.e. dA – DA vs. time) for the pyramids along the main trendline are displayed in Fig. 30. Obviously, the errors in the orientation of Snofru's, Khufu's, Menkaure's, and Neferirkare's pyramids towards the vertical alignment of Kochab and ζ UMi show no secular trend over time and stay at a surprisingly low level. The impressive precision and accuracy of this orientation can be partly explained by a lower position of the target stellar configuration, and by felicitous timing of observations at dead of night and during winter time when the visual effects of turbulent airflows in the Earth atmosphere were reduced.

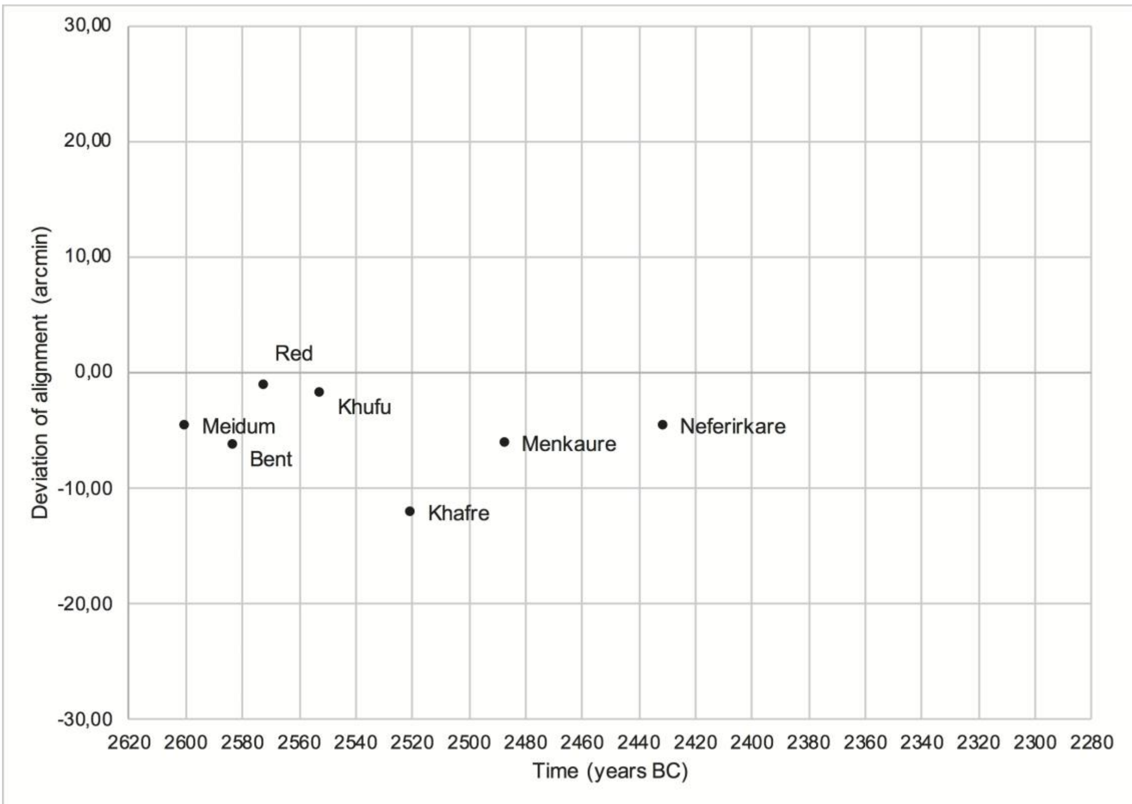

Fig. 30: Precision of pyramid orientations (east sides) towards the lower vertical alignment of Kochab and ζ UMi (von Beckerath's chronology with 48 years for the duration of Snofru's reign).

The pattern of the errors shown in Fig. 30 is very similar to that in the method based upon the orientation towards Mizar at the moment of the horizontal alignment of Mizar and Alioth (Fig. 21). The reason lies in a remarkable astronomical coincidence: in the time of the Old Kingdom, the azimuth of Mizar at the moment of its horizontal alignment with Alioth was very close to the azimuth of the vertical alignment of Kochab and ζ UMi (Fig. 31); both directions coincided around 2562 BC. [87]

[87] That could provide a unique opportunity to orient astronomically both the east and west sides of the Red and Khufu's pyramids in the course of one night.

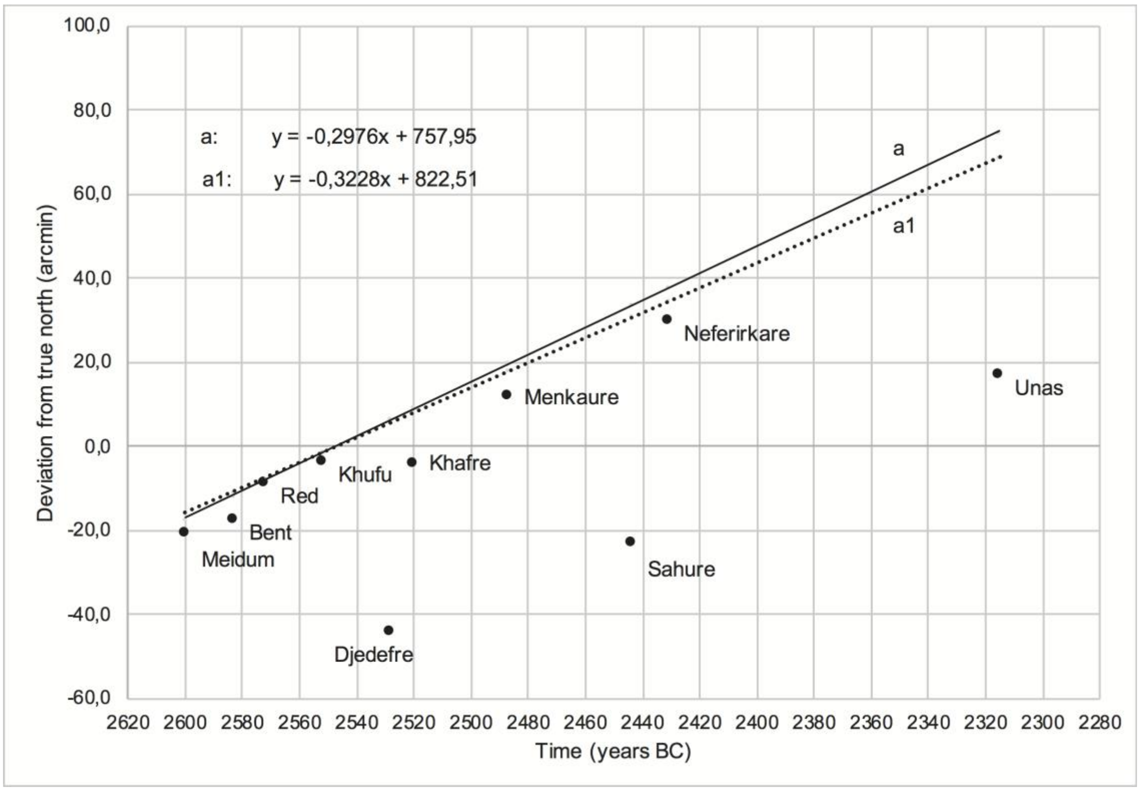

Fig. 31: Deviation of pyramid orientations (east sides) from true north over time (von Beckerath's chronology) vs. azimuths of Mizar at the moment of the upper horizontal alignments of Mizar and Alioth (trendline 'a1'), and vs. azimuths of the lower vertical alignment of Kochab and ζ UMi (the dashed line 'a').

Thus, one cannot unambiguously conclude which of the two methods was used in the orientation of pyramids. The method of orientation towards the vertical alignments of stars had, however, some technical advantages, and we consider it to be more applicable.[88] In this case, the second trendline in the orientation of the pyramids can be explained as targeting Mizar at the moment of vertical alignment of Kochab and ζ UMi. The trendline 'f' at Fig. 29 is drawn through the azimuths of Mizar at the corresponding points in time; the mean value of the differences between the azimuths of Mizar and the azimuths of the east sides of Djedefre's, Sahure's, and Unas's pyramids is about 11´for von Beckerath's chronology with a standard deviation of 2.3´. In fact, this error is very small for naked-eye observations (let us recall that the apparent diameter of the Sun is about 30´). Provided that the results of surveying these badly-preserved pyramids are reliable, such an error can be explained by the more complicated problem of targeting a single star and by the high altitude of Mizar over the horizon at the times of the vertical alignments of Kochab and ζ UMi; as was discussed in the section "Alignments and Errors", the breadth of the construct used for observation of a star at high altitude leads to an uncertainty in finding its direction. One cannot, however, exclude the possibility that Djedefre's, Sahure's, and Unas's pyramids were oriented with the help of another stellar alignment and the hints of this alternative variant may be found in future studies by examining data for the orientation of other ancient Egyptian monuments.

Thus far, we have considered the orientation of the *east* sides of the pyramids towards the different stellar configurations. For the vertical alignment of Kochab and ζ UMi, we will also give the results of our calculations for the west sides. As shown in Fig. 32, the precision of

[88] We assume also that the preserved Egyptian astronomical diagrams are rather consistent with the method based upon the vertical alignment of stars.

this orientation is very impressive and cannot be questioned for the Meidum, Bent and Khufu's pyramids. The only visible deviation from the trendline 'a' is for Menkaure's pyramid. One should take into account, however, that because the foundation of the pyramid was covered with rubble, Nell and Ruggles (2014) were only able to survey the alignment of courses of stones on the pyramid itself. The results for two block courses (9th and 11th) were given in their Table 5c with orientations of 29.5´ and 19.7´, correspondingly (the figure of 25´ given in our Table 1 is the mean value of these estimations). For the figure of 19.7´, the azimuth of the west side of Menkaure's pyramid would lie exactly on the line 'a'; this position is marked at Fig. 32 with a cross.[89] In our opinion, although the data for the west sides of the pyramids are sparse, these calculations strongly support the idea that the Stretching of the Cord ceremonies started with the west sides and, as discussed in Appendix C, usually with the north-west corner.[90]

For the pyramids along the secondary trendline, only two measurements for the west sides are available. The orientation of the west side of Unas' pyramid towards Mizar at the time of the vertical alignment of Kochab and ζ UMi has an error of 10´ and that of Djedefre's pyramid an error of 6´ (let us recall that the azimuth of the latter pyramid is known only with a precision of ±10´).

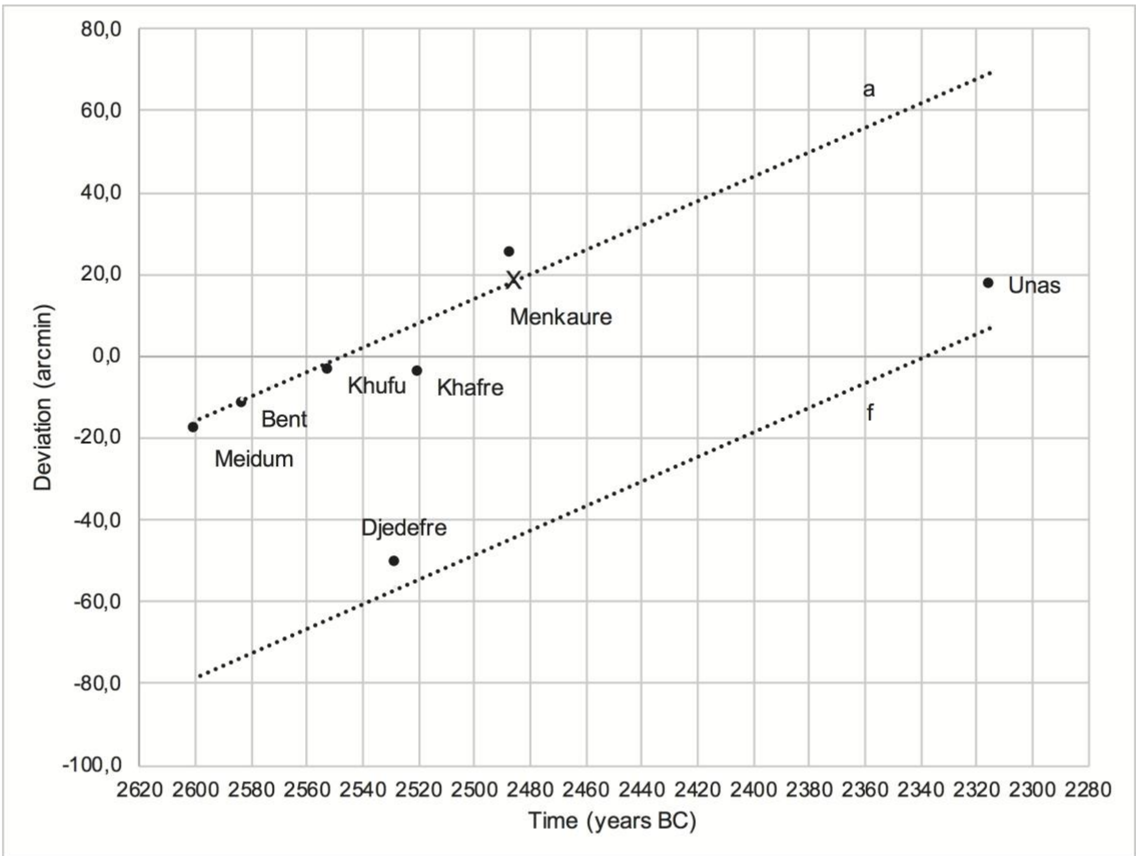

Fig. 32: Deviations of pyramid orientations (west sides) from true north over time vs. azimuth of the lower vertical alignment of Kochab and ζ UMi (von Beckerath's chronology). The dashed line 'a' is a trendline through the azimuths of this vertical, and the dashed line 'f' is the trendline through the azimuths of Mizar at the time of this vertical alignment.

[89] On what grounds this change in the orientation was made is not clear. Did the ancient architects notice that in the process of the construction the initial orientation was distorted, and try to correct it? If so, the initial orientation was close to the orientation of the 11th course. The corresponding inner angle of the north-west side of the pyramid would also best match the right angle (see Appendix C).

[90] See also Spence 2000, 321.

We have also checked our new proposal vs. Baines and Malek's chronology (1980)[91] with the years of the construction for the Bent and Red pyramids chosen according to the proportion used by K. Spence. Because the dates for the pyramids along the main trendline for both chronologies are very close, the resulting precision of the orientation towards the vertical alignment of Kochab and ζ UMi is also similar. As shown in Fig. 33, although the gradients of the lines 'a' and 'c' differ in the case of Baines and Malek's chronology more than in the case of von Beckerath's dates, a very good match still exists[92] and a choice between the two variants cannot be easily made: the mean deviation for the pyramids along the main trendline from the azimuths of the stellar vertical now becomes -4.4´with a standard deviation of about 2.6´.

In our opinion, these results make any chronological correction unsubstantiated, and our calculations strongly confirm Baines and Malek (1980) or von Beckerath's low chronology for the dates of the 4th Dynasty with Stadelmann's modification for 48 years of Snofru's reign.

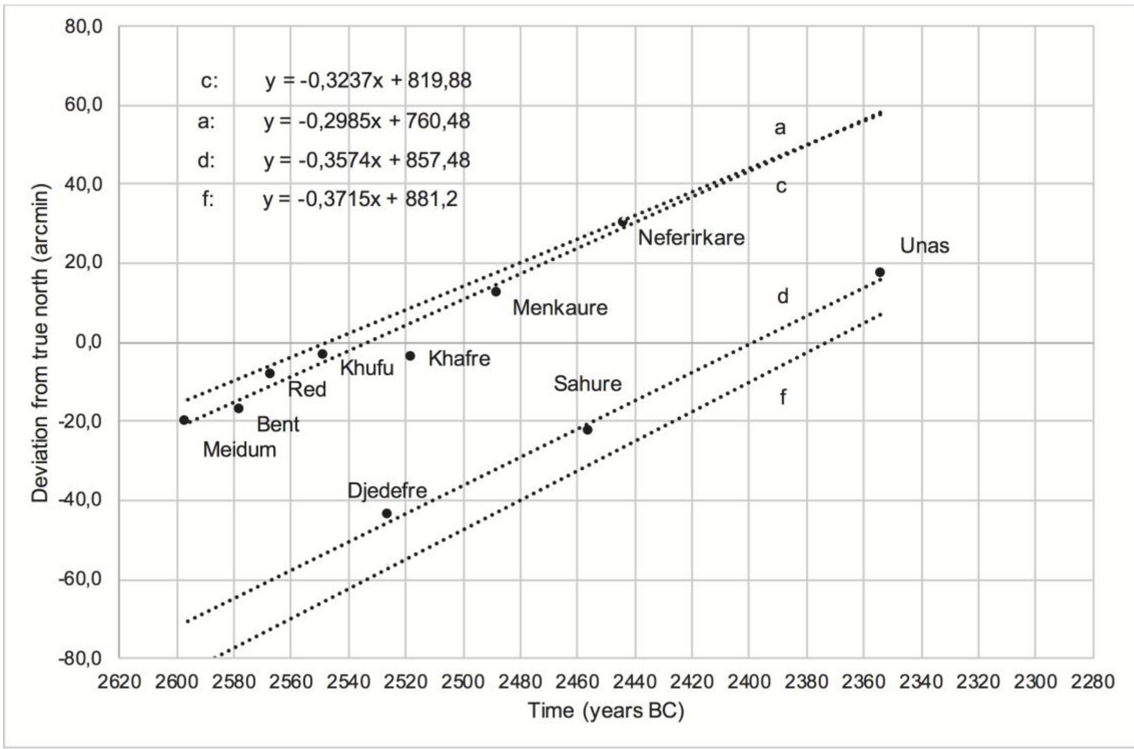

Fig. 33: Deviation of pyramid orientations (east sides) from true north over time vs. azimuth of the lower vertical alignment of Kochab and ζ UMi (Baines and Malek's chronology). The dashed line 'a' is a trendline through the azimuths of this vertical and the dashed line 'f' is the trendline through the azimuths of Mizar at the time of this vertical alignment.

At this point, one should pose the question of whether a fine astronomical tuning can finally solve the problem of the duration of Snofru's reign. Could a range of 29 to 48 years, allowed according to current debates, significantly change our results? Clearly, the gradients of the main trendlines calculated in this text will become slightly different, but for the graphs displaying the precision of the orientations of the pyramids to the selected targets, only the positions of the three points corresponding to the azimuths of Snofru's pyramids will change,

[91] Once more, we follow here Stadelmann (1986, 229–240) and add Huni's 24 years to Snofru's reign.
[92] The 13-years difference between the accession dates of Neferirkare is mainly responsible for the change in the gradients of the main trendlines; it means, however, that Neferirkare's pyramid would be better adjusted to the vertical alignment of Kochab and ζ UMi using Baines and Malek's dates. Such a development may indicate an increase in precision of the observations and methods of orientation.

and our conclusions remain valid. However, the pattern of the errors let us surmise that the duration of the 48 years for Snofru's reign may be preferable. Fig. 34 displays the errors in the orientation of the pyramids towards the vertical alignment of Kochab and ζ UMi for von Beckerath's chronology modified by 30 years for Snofru's reign as proposed by R. Krauss (1996). Comparison with Fig. 30 shows that the 'compression' of the possible dates of the alignment ceremonies change the regular pattern of the systematic errors (although they stay at a very low level) and—if accepted—requires an additional explanation.

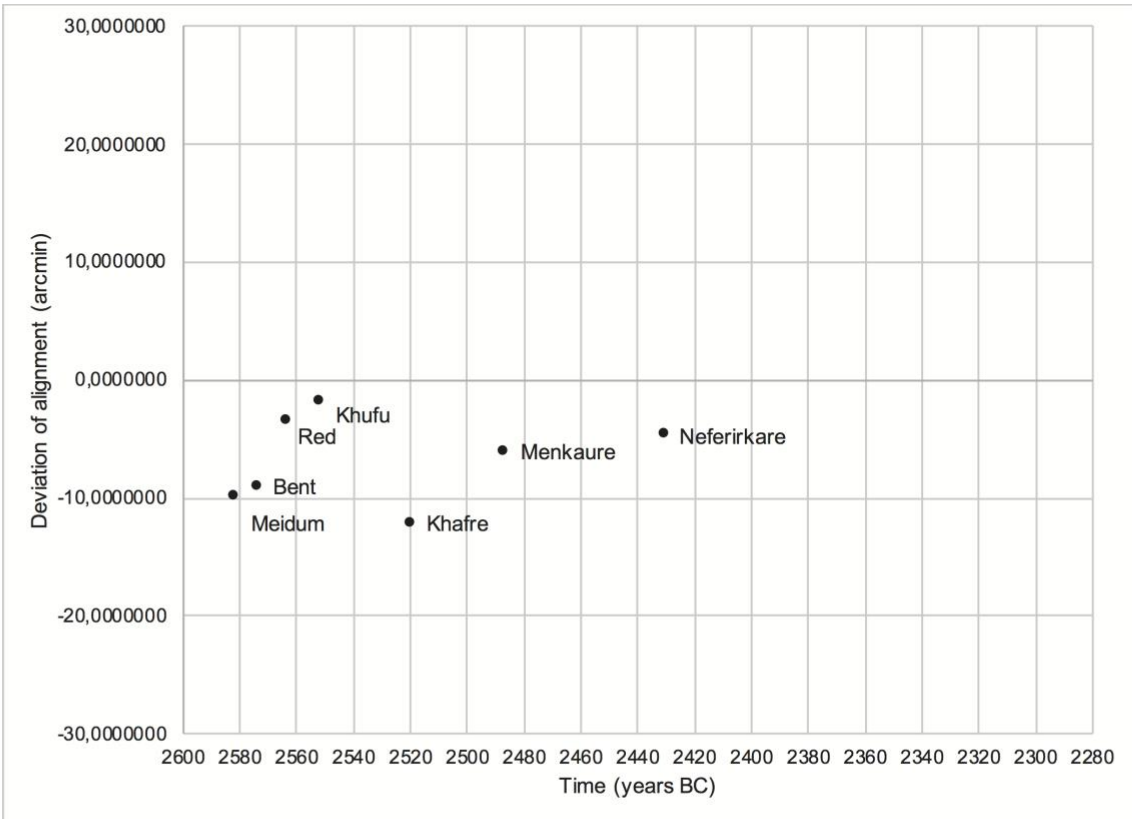

Fig. 34: Precision of pyramid orientations (east sides) towards the lower vertical alignment of Kochab and ζ UMi (von Beckerath's chronology with 30 years for the duration of Snofru's reign).

To conclude, comparison between our results and Belmonte's (Figs. 9–12) and Puchkov's proposals (Figs. 17–18) clearly shows that our method matches the azimuths of the pyramids along the main trendline with a higher precision and consistency. An important advantage of our proposal is that the vertical alignment of Kochab and ζ UMi could have been observed at a lower altitude, and this fact makes our method the only solution advanced so far which eliminates the problem of the high-altitude observations. It should be noted that the vertical alignments of different stars at the lower positions can cause both increases or decreases in their azimuths over time. Whereas Belmonte's proposal results in *decreasing* azimuths of lower vertical alignments of Phecda and Megrez and is thus opposite to the main trendline 'c', our proposal results in *increasing* azimuths of the lower vertical alignments of Kochab and ζ UMi and matches this trendline. The simple mathematical model of precession discussed in Appendix B explains this counterintuitive result.

Our proposal is shown schematically in Fig. 35 where the *Cartes du Ciel* image (Fig. 26) is projected on the northern panel of Senenmut's astronomical diagram (Fig. 22). It should be noted that the asterism Little Dipper in such a position lies near the horizon (the altitude of the Polaris, α Umi, is only about 6°) and that even slightly hilly ground obscures the stars of the ‘handle’ of the Adze and makes the asterism appear as two parallel stellar arrangements.

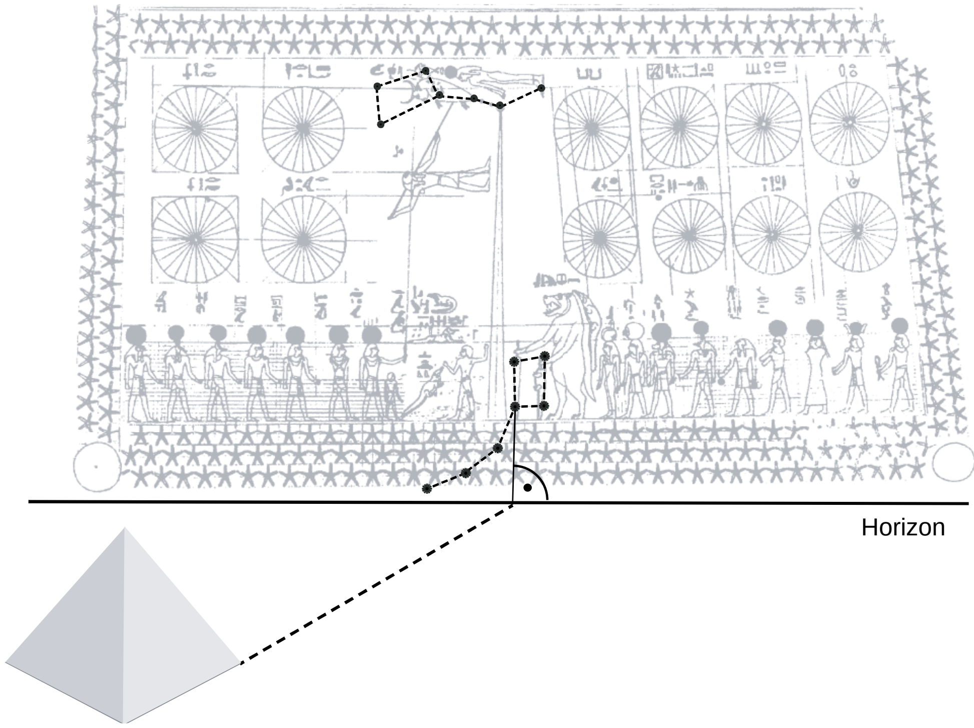

Fig. 35: Symbolic interpretation of the observation of the vertical alignment of Kochab and ζ UMi against the northern panel of Senenmut’s astronomical diagram (not to scale).

The new proposal allows a re-assessment of the problem of the alignment of Djoser pyramid with its deviation from true north of about 180 arcmin. So far, we have neglected this problem in our discussion in order to increase the resolution of the figures (the digression of Djoser’s pyramid from true north falls outside the axes of our graphs) but now we can propose a target direction: Djoser’s pyramid was oriented by the ‘star-architect’ Imhotep towards the second *Mooring Post*—to the vertical alignment of γ UMi (Pherkad) and η UMi which happened in 2640 BC at the azimuth of 186 arcmin. The precision of this orientation—6´—is close to the precision for the later pyramids of Snofru, Menkaure and Neferirkare (Table 5).[93]

Another observation which might have had a symbolic meaning at the time of the Old Kingdom is that at the moment of the vertical alignment of Kochab and ζ UMi, a very important star *Sepedet*/Sothis/Sirius, mentioned in the Pyramid Texts in direct connection with the *Meshketiu,* was setting in the western part of the horizon:[94]

> The sky has been bled and Sothis lives, for Unis is the living one, Sothis’s son, for whom the Dual Enneads have cleaned the imperishable Striker [Meshketiu]. Unis’s house for the sky will not perish, Unis’s seat for the earth will not end.

[93] Although we prefer to restrict our calculations to the data for the reliably measured pyramids, we should mention that for our proposal, the orientation of Sekhemkhet’s and Khaba’s pyramids can be explained as oriented towards the star Polaris at the moment of the vertical alignment of Kochab and ζ UMi and that of γ UMi (Pherkad) and η UMi, respectively.

[94] The Pyramid Texts of Unas, Spell 207 (Allen 2005, 54).

Five minutes later, by the time of the vertical alignments of another side of the Little Dipper—γ and η of UMi—Sirius was already under the horizon. The position of Sirius at the moment of the vertical alignment of Kochab and ζ UMi is displayed in Fig. 36.

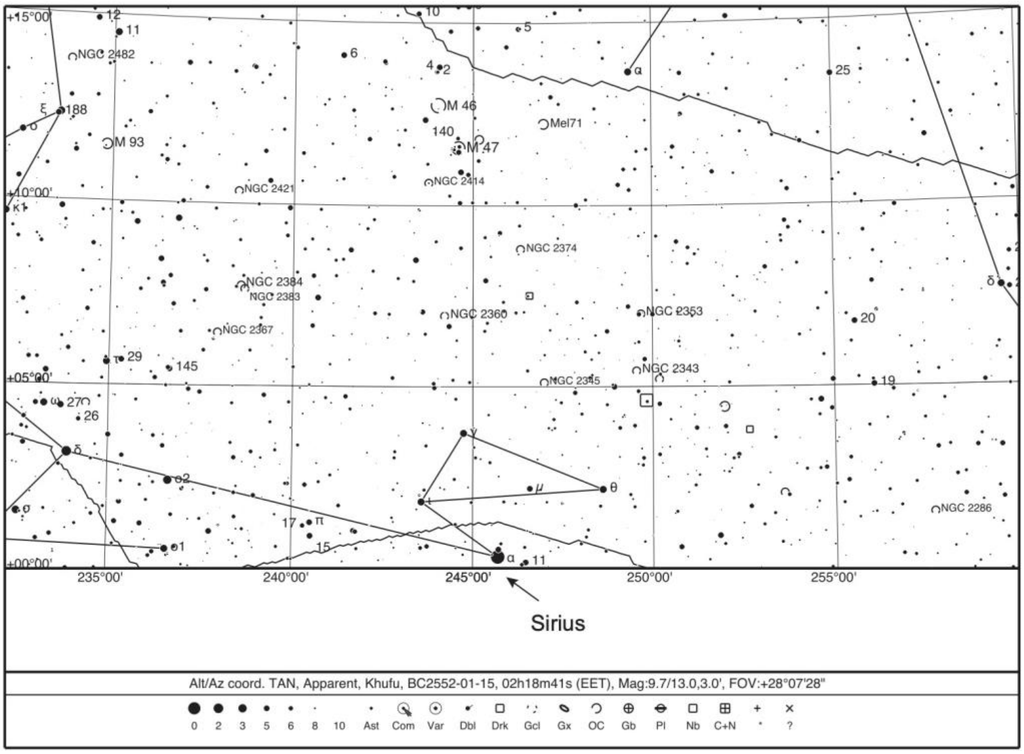

Fig. 36: Position of Sirius at the moment of the vertical alignment of Kochab and ζ UMi at the latitude of Khufu's pyramid on January 15, 2552 BC. Map produced on *Cartes du Ciel*.

Conclusion

The principal distinction in our approach to the problem of the temporal trend in the orientation of the Old Kingdom pyramids is of an epistemological character: whereas the majority of previous scholars assumed the intended orientation of the pyramids to be the North Celestial Pole and looked for the stars which could have helped to mark its position at the time in question, we assume instead that the ancient architects were looking for remarkable stars in remarkable geometrical stellar configurations. The numerous examples of identification of the dead king with a star are given in the Pyramid Texts[95] but no textual evidence identifies a king directly with the Pole.

Because of the diurnal rotation of the celestial sphere, the target of observation could only be fixed and marked by a notable stellar configuration; for some of the ancient architects, this stellar configuration could have meant a vertical or horizontal alignment of stars. Spence (2000) proposed that the architects of the 4th Dynasty found true north by observing two stars, Kochab and Mizar, aligned vertically close to the meridian at the time of the pyramid foundation ceremonies; shortly afterwards, J. Belomonte (2001) suggested, on the same ground, the vertical alignment of Phecda and Megrez. Both methods were aimed to explain the orientation of the pyramids of Snofru, Khufu, Menkaure and Neferirkare (the main trendline in the

[95] See, e.g. Faulkner (1969); Allen (2005).

azimuths); the orientation of Djedefre's, Sahure's, and Unas's pyramids cannot be explained with the help of the same target stellar configuration. Both methods demand observations at high altitudes and have a problem of an apparent lack of corresponding astronomical instruments. Recently, A. Puchkov (2019) proposed that the orientation of the pyramids was to different stars at the moment of the horizontal alignment of Dubhe and Alkaid; the orientation of the pyramids along the main trendline is explained by that author by the orientation towards Thuban (because of its proximity to the North Celestial Pole), and the orientation of Djedefre's, Sahure's, and Unas's pyramids by their orientation towards Alioth. Although Thuban could have been observed at a relatively low altitude of about 30°, the horizontal alignment of Dubhe and Alkaid as well as the positions of other stars would need to have been observed at high altitudes, and the method has a problem similar to that of K. Spence's and J. Belmonte's procedures.

All these publications considered the astronomical data to be known with great precision and treated existing Egyptian chronologies of this period as only relative. Accordingly, the discrepancy between the azimuths of the sides of the pyramids and the azimuths of the corresponding stars was interpreted as a consequence of erroneous historical dating and the astronomical data were used to anchor the archeological data in time.

Whereas K. Spence (2000: 324) proposed shifting von Beckerath's lower chronology by about 74 years to the past (this shift depends on many parameters discussed in our text), the latest proposal of A. Puchkov demands "a shift in the dates of the Old Kingdom by a little more than two centuries to the past" (Puchkov 2019: 19). In contrast, the method advanced by J. A. Belmonte does not demand a significant change of the historical dating and proposes for Khufu's ascension to the throne the dates "just in the middle of the highest and lowest chronologies accepted by Egyptologists" (Belomonte 2001: S12).

Underlying all these proposed changes of chronology was a silent assumption that the errors in the orientation of the pyramids towards the target of observations were negligible, and no attempt was undertaken by the authors to calculate the errors that may arise when performing such an alignment. We have considered the aforementioned proposals and analyzed for each variant the corresponding errors, both random and systematic, against some conventional chronologies of the Old Kingdom.

It was shown that the errors in J. A. Belmonte's hypothesis show a noticeable secular trend over time: for Shaw's chronology, precision (with a mean error of 8.5´and standard deviation of 5.5´) in the orientation of the pyramids towards the vertical alignment of Phecda and Megrez increases over time between the dates of the alignments of the Meidum and Neferirkare's pyramids, which makes the apparent accuracy of the northern orientation of Khufu's pyramid purely fortuitous (Figs. 9–10), whereas for von Beckerath's chronology, the precision (with a mean error of -6.5´and standard deviation of 6.6') in the orientation towards this stellar alignment steadily decreases (Figs. 11–12). This decreasing precision in the adjustment of the pyramids over time is, however, difficult to explain.

Two other methods minimize the mean errors in the orientation of pyramids towards the target constellation by a selected shift in time. Our analysis of the remaining errors reveals that in Puchkov's solution, a secular trend also exists and shows a similar decreasing precision in the orientation towards Thuban (Figs. 17–18). In contrast, Spence's solution does not show a noticeable secular trend in the precision of the orientation of the pyramids to the vertical alignment of Kochab and Mizar for von Beckerath's chronology, as well as for the (modified by 74 years) chronology proposed by the author (Figs. 4 and 6). Because the azimuth of a vertical alignment of two stars depends upon their right ascensions and declinations, the similar trends between the azimuths of the sides of the pyramids and the azimuths of vertical alignment of Kochab and Mizar points towards at least one of these two stars having been used in the pyramid orientation ceremonies.

We propose in this text two new alternative methods to explain the temporal trend in the orientation of the pyramids, one based upon the horizontal alignment of Alioth and Mizar, and another upon the vertical alignment of Kochab and ζ UMi. In the time of the Old Kingdom, these two astronomical events might have been observed in close temporal proximity: provided that the direction towards Mizar at the moment of its horizontal alignment with Alioth was fixed, about 16 minutes later almost the same direction was marked by the vertical alignment of Kochab and ζ UMi; the two directions practically coincided in 2562 BC (Fig. 31).[96]

Although the years of the alignment procedures are not known exactly and sometimes are based on educated guesses, our proposals—quite unexpectedly—can serve as a fine tuning for relative chronologies. For both methods, the trendline drawn through the azimuths of the stellar alignments matches impressively well the main trendline in the orientation of the Old Kingdom pyramids for von Beckerath's (1997), and Baines and Malek's (1980) dates for the 4th Dynasty without a noticeable secular difference between the two lines (Figs. 29–30 and 33). For von Beckerath's chronology, the mean error in the orientation of Snofru's, Khufu', Neferirkare' and Menkaure' pyramids towards the vertical alignment of Kochab and ζ UMi attains about -4´ with a standard deviation of only 2´ and is thus close to the limit of naked-eye observations.

Although based on different observations, both methods show similar precision, and one cannot exclude the possibility that both methods were used concomitantly. The pyramids could have been oriented towards Mizar by directly targeting that star at the moment of its horizontal alignment with Alioth (first method), or towards the vertical alignment of Kochab and ζ UMi as a convenient alternative solution to reach the realm of the *Imperishable Stars* (second method). An important advantage of the second method is that this vertical alignment might have been observed at lower altitude, with Kochab at only about 20° over the horizon. At such an altitude, a standard old Egyptian astronomical instrument, a *merkhet*[97] supplied with a plumb line, could approximately determine the moment of this vertical alignment, and the posts used in the Stretching of the Cords ceremony might have been used to additionally check the alignment and to fix the direction towards this stellar configuration on the ground. Of course, the evidence discussed in this text is not a strong mathematical proof, and the comparison of the graphs cannot serve as a final argument to reject the methods proposed by other authors. But our results match the main trendline in orientation of the pyramids with such a compelling precision for the existing and widely agreed upon chronologies that the probability of a chance match is approximately the same as the probability of a chance temporal trend in the orientation of the pyramids.

Our approach makes it possible to explain the orientation of three pyramids which do not follow the famous pattern of the main trendline—Djedefre's, Sahure's and Unas'—as a result of complementary targeting methods, i.e. as adjustment to Kochab or ζ UMi at the moment of horizontal alignment of Alioth and Mizar or, alternatively, as orientation towards Mizar at the moment of the vertical through Kochab and ζ UMi. We consider the second possibility to be more logical and explain the slightly lower precision of such an orientation in the range of 6–11´ by the more complicated problem of targeting Mizar at its high altitude.

The results of our calculations let us surmise that the *Mooring Posts* in the astronomical diagrams can be interpreted as the sides of the Little Dipper constellation at the moment when they were visually aligned vertically as shown in Fig. 35. We assume that the star encircled in red in the constellation *Meskhetiu* in the northern panel of Senenmut's astronomical diagram

[96] Remarkably, in 2563–2562 BC, the same direction was also marked by the vertical alignment of Phecda and Megrez (Fig. 11).

[97] Two instruments preserved in the Egyptian Museum of Berlin have a different size but are marked according to the same scheme (although not very accurately). We can, therefore, assume that the size of the *merkhet* was not standardized and that bigger instruments possibly also existed.

(displayed as a target of observation) might be identical with Mizar and that this star was the intended target of the alignment of the pyramids.

It is tempting to interpret the term *aqA* of *Meskhetiu* (mentioned e.g. in the inscriptions in the Temple of Hathor at Dendera) as the position of this asterism (the Big Dipper) at the moment of the vertical alignment of Kochab and ζ UMi in the Little Dipper. In this case, the preserved iconographic evidence (such as Senenmut's astronomical diagram, ceiling details in the Temple of Ramses II or in the tomb of Seti I) display *exactly* the *aqA* position of *Meskhetiu* and may need to be reconsidered in view of this possibility.

The azimuths of the alignments of stars depend not only upon their right ascensions and declinations, but also upon the geographical position of observation. That the alignments occured at the time of the construction of Snofru and Khufu's pyramids close to true north is, in our opinion, a fortuitous event which is mainly responsible for the remarkable northern orientation of these pyramids. It can be explained in a simple way: they were just constructed at the right place and at the right time.

Appendix A. Azimuth of Two Vertically Aligned Stars

Let us consider two stars, S_1 and S_2, with equatorial coordinates known for the date of observation as (α_1, δ_1) and (α_2, δ_2), correspondingly. Both stars are assumed to be observed at the known latitude φ along the same vertical thus having the same azimuth. Fig. A1 displays the situation when both stars are aligned vertically at their lower position to the west of the meridian.

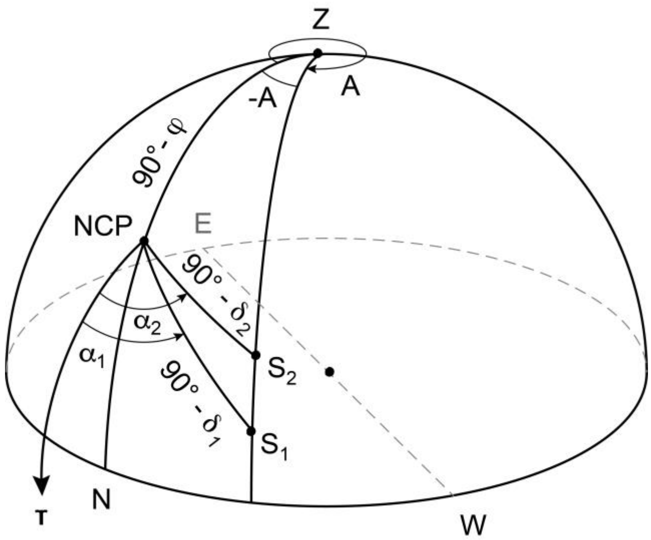

Fig. 1A: Vertical alignment of two stars, S_1 and S_2 to the west from the meridian of an observer. The zenith is marked with Z, the north celestial pole with NCP, and A is the azimuth of both stars.

We will label the stars in such a way that $\alpha_2 > \alpha_1$, then, the azimuth A (reckoned from north towards the east) of their vertical alignment can be calculated with the following formulae:

$$\sin(\mathrm{A}) = \pm\frac{\cos(\delta_1)\cos(\delta_2)\sin(\alpha_2 - \alpha_1)}{\cos\ (\varphi)\sqrt{1-\eta^2}}, \qquad \text{(A1)}$$

where

$$\eta = \cos\ (\widehat{S_1 S_2}) = \sin(\delta_1)\sin(\delta_2) + \cos(\delta_1)\cos(\delta_2)\cos(\alpha_2 - \alpha_1). \qquad \text{(A2)}$$

Here the plus/minus corresponds to the stellar alignment in the east/west, respectively.

Appendix B. Azimuth Variation of Two Vertically Aligned Stars due to Precession

The precession of the equator (described by an angle ψ_A) results from the torques that the Sun, Moon, and planets exert on the equatorial bulge of the Earth; the period of this effect is about 25,780 years. The precession of the ecliptic (described by χ_A) results from the gravitational perturbations of other planets onto the orbit of the Earth-Moon barycenter about the Sun; it causes the variation of the obliquity of the ecliptic between 21°55' and 24°18'. The general precession (accumulated effect of precession of equator and ecliptic) is usually described by three precession angles given as polynomials expressed over time (Lieske et al. 1977). The rate of the change in azimuths of stellar alignments should be calculated with the help of the formulae representing these effects. The direction of this change, however, can be estimated in some cases in a simple way.

We consider two stars labelled with the indices “1” and “2” aligned vertically on one side of the Pole close to the meridian, i.e. $|A| \ll 1$ and $|\alpha_2 - \alpha_1| \ll 1$ with $\alpha_2 > \alpha_1$ where the angles are expressed in radians. For short time span T, we will approximate the effect of precession by its linear part, so that the right ascensions and declinations are given as

$$\alpha(\mathrm{T}) = \alpha^0 + \alpha' \,\mathrm{T}, \;\; \delta(\mathrm{T}) = \delta^0 + \delta' \,\mathrm{T}.$$

Here

$$\alpha' = \mathrm{m} + \mathrm{n} \sin\alpha \tan\delta, \qquad \delta' = \mathrm{n}\cos\alpha,$$

$$\mathrm{m} = \mathrm{d}\psi_\mathrm{A}/\mathrm{dt}\, \cos\epsilon - \mathrm{d}\chi_\mathrm{A}/\mathrm{dt}, \;\; \mathrm{n} = \mathrm{d}\psi_\mathrm{A}/\mathrm{dt}\, \sin\epsilon,$$

where ϵ is the mean obliquity of the ecliptic, T is expressed in centuries and the contemporary figures of the coefficients are $\mathrm{d}\psi_\mathrm{A}/\mathrm{dt} \cong 5039''/\mathrm{century}$ and $\mathrm{d}\chi_\mathrm{A}/\mathrm{dt} \cong 11''/\mathrm{century}$ (Lieske et al. 1977).
In our approximation, the precession in δ can be ignored for $\mathrm{dA/dt}$ and

$$\alpha_2 - \alpha_1 = \alpha_2^0 - \alpha_1^0 + (\alpha_2' - \alpha_2')\mathrm{T} =$$
$$= \alpha_2^0 - \alpha_1^0 + \mathrm{d}\psi_\mathrm{A}/\mathrm{dt}\, \sin\epsilon\, (\sin\alpha_2 \tan\delta_2 - \sin\alpha_1 \tan\delta_1)\, T.$$

Substituting this estimation in formula A1, using the approximation sin(A) ≅ A, $\sin(\alpha_2\text{-}\alpha_1) \cong \alpha_2 - \alpha_1$, and ignoring the effects of the second order we obtain

$$\mathrm{dA/dt} = \pm k\, (\sin\alpha_2 \tan\delta_2 - \sin\alpha_1 \tan\delta_1) \equiv \pm k \cdot \mathrm{d}, \tag{B1}$$

where the plus/minus corresponds to the alignment in the east/west direction, and the coefficient

$$k = \mathrm{d}\psi_\mathrm{A}/\mathrm{dt}\, \sin\varepsilon \frac{\cos(\delta_1)\cos(\delta_2)}{\cos\,(\varphi)\sqrt{1-\eta^2}} \tag{B2}$$

is positive for circumpolar stars and an observer in Egypt. Formula B1 shows that an increase or decrease in azimuth depends upon the sign of the expression $\pm$d, i.e. upon the position of the alignment (+ to the east and − to the west) and upon the right ascensions and declinations of stars; it can be different for different pairs of stars at the moments of their vertical alignments.

Formula B1 explains the following counter-intuitive result: whereas the azimuth of the vertical alignment of Phecda and Megrez decreases over time, the azimuth of the vertical alignment of Kochab and ζ UMi increases when both pairs are at their lower alignment.[98]

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[98] Appendix B is written in collaboration with M. Soffel.

Appendix C. Interior Angles of Pyramid Foundations

For the orientation data given in Table 1, one can calculate the interior angles of the pyramids foundations; due to the limited number of measurements and different readings for the course blocks and casing stones, the following estimations can be considered only as preliminary results. Using notations of Fig. C1 we obtain:
For Meidum pyramid:

$\angle\alpha = 90° - dA_W + dA_N = 90° - 18.1' + 35.4' = 90°\,17.3'$,
$\angle\beta = 90° - 35.4.' + 20.6' = 89°\,45.2'$,
$\angle\gamma = 90° - 20.6' + 23.6' = 90°\,3'$,
$\angle\delta = 90° - 23.6' + 18.1' = 89°\,54.5'$.

For Bent pyramid:

$\angle\alpha = 90° - 11.8' + 7.5' = 89°\,55.7'$,
$\angle\beta = 90° - 7.5' + 17.3' = 90°\,9.8'$,
$\angle\gamma = 90° - 17.3' + 4.2' = 89°\,46.9'$,
$\angle\delta = 90° - 4.2' + 11.8' = 90°\,7.6'$.

For Khufu's pyramid:

$\angle\alpha = 90° - 3.7' + 3.6' = 89°\,59.9'$,
$\angle\beta = 90° - 3.6' + 3.4' = 89°\,59.8'$,
$\angle\gamma = 90° - 3.4' + 0.5' = 89°\,57.1'$,
$\angle\delta = 90° - 0.5' + 3.7' = 90°\,3.2'$.

For Djedefre's pyramid:

$\angle\alpha = 90° - 50.8' + 51.7' = 90°\,0.9'$,
$\angle\beta = 90° - 51.7' + 43.9' = 89°\,52.2'$,
$\angle\gamma = 90° - 43.9' + 48.4' = 90°\,4.5'$,
$\angle\delta = 90° - 48.4' + 50.8' = 90°\,2.4'$.

For Menkaure's pyramid with the mean value for the azimuth of the east side of -25´(Nell, and Ruggles 2014, Table 5c):

$\angle\alpha = 90° + 25' - 20' = 90°\,05'$,
$\angle\beta = 90° + 20' - 12' = 90°\,08'$,
$\angle\gamma = 90° + 12' - 32' = 89°\,40'$,
$\angle\delta = 90° + 32' - 25' = 90°\,07'$.

For Menkaure's pyramid with the azimuth of the west side equal to -29.5´ measured at the 9th course (Nell, and Ruggles 2014, Table 5a):

$\angle\alpha = 90° + 29.5' - 20' = 90°\,9.5'$,
$\angle\delta = 90° + 32' - 29.5' = 90°\,2.5'$.

For Menkaure's pyramid with the azimuth of the west side equal to -19.7´ measured at the 11th course (Nell, and Ruggles 2014, Table 5c):

$\angle\alpha = 90° + 19.7' - 20' = 89°\,59.7'$,
$\angle\delta = 90° + 32' - 19.7' = 90°\,12.3'$.

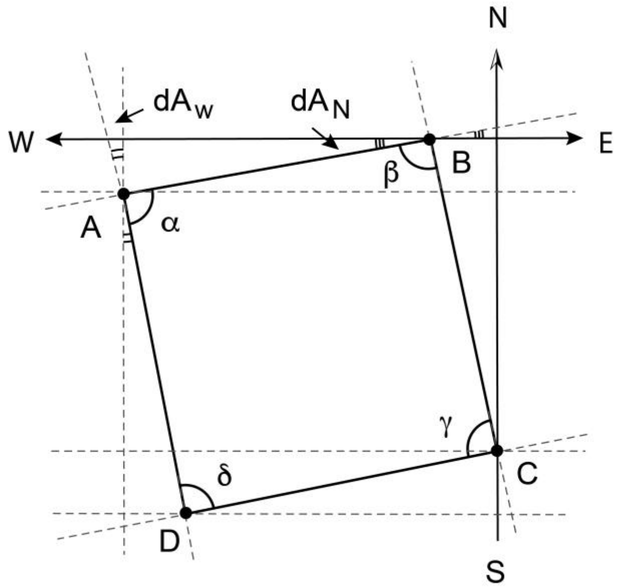

Fig. 1C: Orientation of pyramids and interior angles of foundations.

Obviously, we should assume that the intended interior angles were the right angles. The calculations given above allow us to conclude that for Meidum's pyramid (where only the old measurements of Petrie are available) the 'best constructed' right angle γ lay at the south-east corner, for the Bent, Khufu's, Djedefre's and Menkaure's[99] pyramids the most accurate right angle α lay at the north-west corner. It is especially remarkable in the case of Khufu's pyramid where the angles δ and γ are worst constructed although the included south side is (by chance) best aligned to the cardinality.[100]

The simple explanation for this situation is as follows: we assume that for the pyramids in question it was the west side that was primarily aligned and the length of the pyramid side was staked off along this line. Thus, two 'corners of the pyramid' (A and D at Fig C1) were fixed. Then, a right angle (α) was constructed at point A and a line was drawn towards the north-east point B. The last corner of the pyramid, C, was obtained by adjusting two lengths, BC and DC. Possibly, achieving the same length of pyramid sides was considered to be more important (or easier to realize) than the preservation of right angles.

Acknowledgments

The author is very grateful to V. Yershov for discussing the problems of astronomical observations. Constructive dialog with A. Puchkov has helped to better understand the Egyptological aspects of the problem and to improve the argument of this paper.

[99] For Menkaure's pyramid, the changed orientation of the 11th course at the west side means that the north-west inner angle of the foundation is the most accurately constructed.
[100] The complicated case of Khafre's pyramid will not be considered here. Provided that we have here a "copy and paste" case, the orientation of this pyramid was realized by purely geometrical means.

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