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In discussions of the possible connections between Copernicus and his Islamic predecessors, the Ṭūsī-couple has often been invoked by both supporters and detractors of the actuality of this transmission. But as I maintained in an earlier article, the Ṭūsī-couple, as well as other mathematical devices invented by Islamic astronomers to deal with irregular celestial motions in Ptolemaic astronomy, may be of secondary importance when considering the overall significance of Islamic astronomy and natural philosophy in the bringing forth of Copernican heliocentrism.[1] Nevertheless, the development and use of Ṭūsī's astronomical devices does provide us with important evidence regarding the transmission of astronomical models and lessons about intercultural scientific transmission. So in this article, I will attempt to summarize what we know about that transmission, beginning with the first diffusion from Azerbaijan in Iran to Byzantium and continuing to the sixteenth century. Though there are still many gaps in our knowledge, I will maintain, based on the evidence, that intercultural transmission is more compelling as an explanation than an assumption of independent and parallel discovery.

It will be helpful if we first analyze what exactly is meant by the "Ṭūsī-couple." The first thing to notice is that the term "Ṭūsī-couple" does not refer to a single device or model but actually encompasses several different mathematical devices that were used for different purposes (see Table 1). Because this understanding is not always maintained in the modern literature, there has been considerable divergence, often leading to confusion, about what exactly the Ṭūsī-couple is. This in turn has made it difficult to trace transmission. So a quick historical overview is in order.[2]

[1] F. Jamil Ragep, "Copernicus and His Islamic Predecessors: Some Historical Remarks," *History of Science* 45 (2007): 65–81.
[2] Here we need to acknowledge Mario di Bono, who, in a valuable article, insists on distinguishing the various versions of the Ṭūsī-couple; see his "Copernicus, Amico, Fracastoro and Ṭūsī's Device: Observations on the Use and Transmission of a Model," *Journal for the History of Astronomy* 26, no. 2 (1995): 133-154.

TABLE 1

Name of Device (Ragep)	Description	Used for	Other Names
Mathematical Rectilinear Version	Two uniformly rotating circles, the smaller internally tangent to the larger, to produce rectilinear oscillation	Replacing equant in planetary motions	Plane Version (Saliba/Kennedy); the spherical version with parallel axes and radii in the ratio of 1:2 (di Bono); the device (*aṣl*) of the large and the small [circles][3]; 2 unequal circle version
Physicalized Rectilinear Version	Three solid spheres based on the mathematical version for producing rectilinear oscillation	A substitute for the equant device in planetary models	The physicalized 2 circle version with maintaining sphere
Two Equal Circle Version	Mathematically equivalent to the Rectilinear Version	Intended to account for Ptolemaic motions needing curvilinear oscillation on a great circle arc (but actually produces oscillation on a chord)	the plane version with equal radii (di Bono, pp. 137-8); the pseudo-curvilinear version
The 2-sphere curvilinear version	Truncated Version of the full 3-sphere version		
The 3-sphere curvilinear version	Three concentric spheres, one inside the other, rotating uniformly	Intended to account for Ptolemaic motions needing curvilinear oscillation on a great circle arc (works with a minor distortion)	Spherical Version (Saliba/Kennedy)

[3] It was often referred to as such in astronomical texts after Ṭūsī; Ṭūsī himself does not explicitly use this term to refer to the device though it is implied in the terminology he uses in the *Tadhkira* as distinct from the *Ḥall* (see below).

The Mathematical Rectilinear Version = large/small device = the spherical version with parallel axes and radii in the ratio of 1:2 [di Bono, p. 136]

The first version of the Ṭūsī couple was announced by Naṣīr al-Dīn al-Ṭūsī (597 – 672 H / 1201 – 1274 CE) in a Persian astronomical treatise entitled *Risālah-i Muʿīniyya* (the Muʿīniyya treatise), the first edition of which was completed on Thursday, 2 Rajab 632 H / 22 March 1235 CE.[4] Dedicated to the son of the Ismāʿīlī governor of Qūhistān (in the eastern part of modern Iran), the treatise is a typical *hayʾa* (cosmographical) work, one that provides a scientifically-based cosmology covering both the celestial and terrestrial regions. But in presenting the Ptolemaic configuration of the moon's orbs and their motions, Ṭūsī notes that the motion of the epicycle center on the deferent is variable, which is inadmissible according to the accepted rules of celestial physics that dictate that all individual motions of orbs in the celestial realm should be uniform. He goes on to say: "This is a serious doubt with regard to this account [of the model], and as yet no practitioner of the science has ventured anything. Or, if anyone has, it has not reached us." But "there is an elegant way to solve this doubt but it would be inappropriate to introduce it into this short treatise." He then teasingly turns to his patron: "If at some other time the blessed temper of the Prince of Iran, may God multiply his glory, would be so pleased to pursue this problem, concerning that matter a treatment will be forthcoming." In the chapter on the upper planets and Venus, as well as the one for Mercury, he makes a similar claim, namely that he has a solution that will be presented later. In addition to the problem of the irregular motion of the deferent (sometimes referred to as the "equant problem" though it is somewhat different for the moon), Ṭūsī brings up another "doubt" or difficulty, namely that pertaining to motion in latitude, i.e. north or south of the ecliptic. Ptolemy had rather complex models in his *Almagest* and *Planetary Hypotheses* that generated quite a bit of discussion among Islamic astronomers. One of these was Ibn al-Haytham (d. ca. 430 H / 1040 CE), who objected to the lack of physical movers for these models and provided his own in a treatise that is currently not

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[4] On the *Risālah-i Muʿīniyya,* see F. Jamil Ragep, "The Persian Context of the Ṭūsī Couple," in N. Pourjavady and Ž. Vesel (eds.), *Naṣīr al-Dīn al-Ṭūsī: Philosophe et Savant du XIII*[e] *Siècle* (Tehran: Institut français de recherche en Iran/Presses universitaires d'Iran, 2000), pp. 113-130 and idem, *Naṣīr al-Dīn al-Ṭūsī's Memoir on Astronomy (al-Tadhkira fī ʿilm al-hayʾa),* 2 vols. (New York: Springer-Verlag, 1993), 1: 65-66. See also E. S. Kennedy, "Two Persian Astronomical Treatises by Naṣīr al-Dīn al-Ṭūsī," *Centaurus* 21 (1984): 109-120.

extant. However, Ṭūsī refers to it in the *Muʿīniyya* and also notes that it is not entirely satisfactory but, as with his purported models for longitude, eschews any details.[5]

Since Ṭūsī claims to have an elegant solution, one would assume that he would have presented it to his patron in short order. But, as we shall see, he waited almost ten years to present his new models. One clue to the delay could well be over-optimism on the part of the young Naṣīr al-Dīn; he claimed in the *Muʿīniyya* that he had solutions for all the planets, but as it turned out he was never able to solve the complexities of Mercury. Indeed, as an older man many years later, he was to admit in his *al-Tadhkira fī ʿilm al-hayʾa* (Memoir on the science of astronomy): "As for Mercury, it has not yet been possible for me to conceive how it should be done."[6]

The partial solution occurs in a short treatise that was again dedicated to his patron's son, Muʿīn al-Dīn. This work has come to us with a variety of names: the *Dhayl-i Muʿīniyya* (the Appendix to the *Muʿīniyya* [treatise]), the *Ḥall-i mushkilāt-i Muʿīniyya* (Solution to the difficulties of the *Muʿīniyya*), the *Sharḥ-i Muʿīniyya*, and so on.[7] In all cases that I know, the work is explicitly tied to the *Muʿīniyya*, which would lead one to assume that it must have been written a short time after the treatise to which it is appended. This, though, turns out not to be the case. Thanks to the recent discovery in Tashkent of a manuscript witness of the *Dhayl* with a dated colophon, we can now date this treatise, as also the first appearance of the Ṭūsī couple, to 643 H / 1245 CE:

> The treatise is completed. The author, may God elevate his stature on the ascents to the Divine, completed its composition during the first part of Jamādā II, 643 of the Hijra, within the town of Tūn in the garden known as Bāgh Barakah. [=late October 1245].[8]

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[5] The relevant parts of the Persian text discussed in this paragraph, along with translation, are in Ragep, "Persian Context," pp. 123-125.
[6] Ragep, *Naṣīr al-Dīn,* 1: 208.
[7] The name "*Dhayl-i Muʿīniyya*" is found in the only dated manuscript of the text, namely Tashkent, Uzbekistan, al-Biruni Institute of Oriental Studies MS 8990, f. 2a and f. 22b.
[8] Tashkent, al-Biruni Institute of Oriental Studies MS 8990, f. 46a [original foliation]:

As we can infer from the colophon, Ṭūsī was still in the employ of the Ismāʿīlī rulers of Qūhistān in southern Khurāsān. Tūn, present day Firdaws, lay some 80 km/50 miles west-north-west of the main town of the region, Qāʾin, which was the primary regional capital for the Ismāʿīlīs.[9]

It clearly took Naṣīr al-Dīn longer than he anticipated to reach a solution, and even then it was not complete by any means. This "first version" of the Ṭūsī couple consisted of a device composed of two uniformly rotating circles that could produce oscillating straight-line motion in a plane between two points. One of these two circles was twice as large as the second, the smaller being inside the larger and tangent at a point (see Figure 1). The rotation of the smaller was twice that of the larger. Although mathematically speaking the production of an oscillating point on a straight line could also be produced by the small circle "rolling" inside the larger, Ṭūsī is explicit that the larger circle "carries" [*mī bard*] the smaller. The reason for this is that Ṭūsī will transform these circles into the equators of solid orbs rotating in the celestial realm, where any penetration of one solid body by another is expressly forbidden.[10] The transformation into solid orbs, the "Physicalized Rectilinear Version," is shown in Figure 2. Note that one needs a third orb, what he calls the "enclosing sphere [muḥīta] for the epicycle" in order not to disrupt the epicycle. More on this later when we discuss Oresme.

تمت الرسالة اتفق فراغ المصنف رفع الله مراتبه في معارج القدس من تأليفه أوائل جمادى الآخرة سنة ٦٤٣ هجرية بمقام بلدة تون بالبستان المعروف بباغ بركه

[9] On Tūn as one of the residences of the local Ismāʿīlī rulers, see Farhad Daftary, "Dāʿī", *Encyclopaedia Iranica*, ed. Ehsan Yarshater, vol. 6 (New York, 1993), pp. 590-593 on p. 592 (col. 1).
[10] Naṣīr al-Dīn al-Ṭusi, *Ḥall-i mushkilāt-i Muʿīniyya,* facsimile of Tehran, Malik 3503 with an introduction by Muḥammad Taqī Dānish-Pizhūh (Tehran: Intishārāt Dānishgāh Tahrān (no. 304 in the series), 1335 H. Sh./1956-7 A.D.), p. 7:

اما استقامت حركت مركز تدوير از محيط مايل بر سمت مركزش وبعد از آن رجوع او هم بر آن سمت تا بمحيط رسيدن بى آنكه خرقى و التيامى لازم آيد يا خللى باستدارت حركات راه يابذ بر آن وجه تواند بود كه ياد كنيم.

The rectilinear motion of the center of the epicycle away from the circumference of the inclined plane in the direction of its center and its return on that same line until it reaches the circumference occurs without there being any tearing, mending, or rupture in the circular motion.

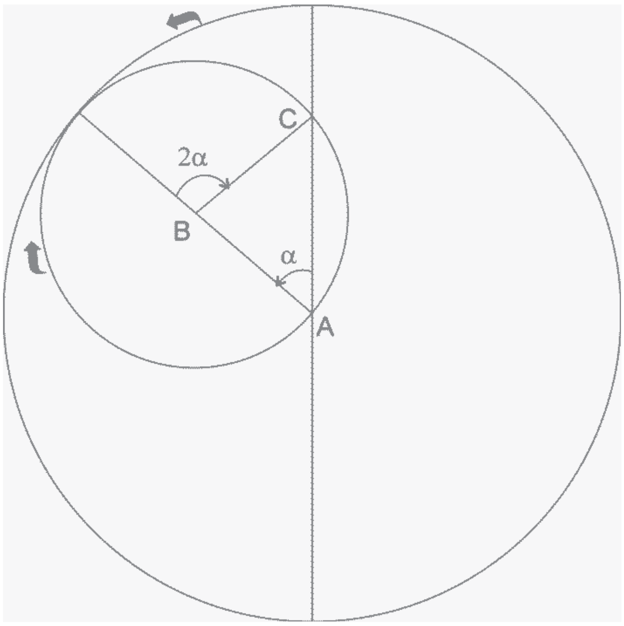

Fig. 1

Mathematical Rectilinear Version

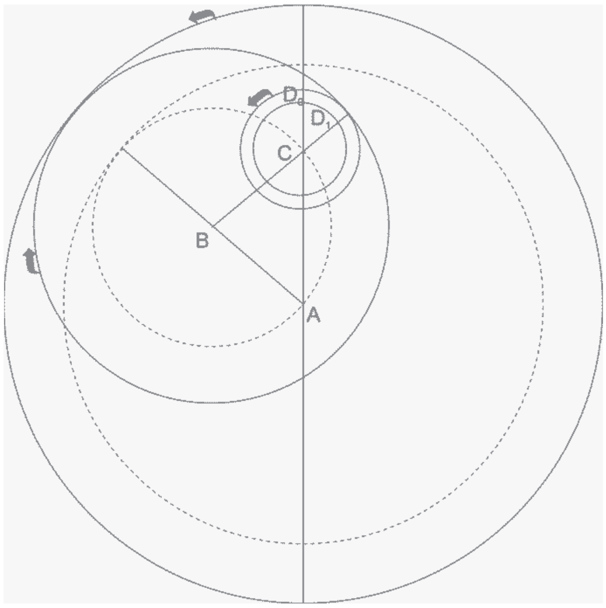

Fig. 2

Physicalized Rectilinear Version

Ṭūsī then proceeds to use the device to construct his alternative to Ptolemy's lunar model. It will be instructive, and important for tracing transmission, to compare this model from the *Ḥall* with the model Ṭūsī would present in his *al-Tadhkira fī ʿilm al-hayʾa*, which unlike the *Muʿīniyya* and *Ḥall,* was written in Arabic. The first version of the *Tadhkira* was completed in 659 H / 1261 CE when Ṭūsī was in the employ of his new patrons, the Mongol Īlkhānid conquerors of Iran. The following chart provides a summary:

Ṭūsī's Lunar Models from the *Ḥall* and *Tadhkira*			
Ḥall		*Tadhkira*	
Orbs	Parameters	Orbs	Parameters
1) Parecliptic Orb (*mumaththal*)	0;3°/day (cs)	1) Parecliptic Orb (*mumaththal*)	0;3°+/day (cs)
2) Inclined Orb (*māʾil*)	13;11°/day (s)	2) Inclined Orb (*māʾil*) 3) Deferent Orb (*ḥāmil*)	11;9°/day (cs) 24;23°/day (s) ------------------ Net: 13;14°/day (s)
3) Dirigent Orb (*mudīr*)	24;23°/day (s) or (cs)	4) Large Sphere (*al-kabīra*)	24;23°/day (s)
4) Epicycle's Deferent Orb (*ḥāmil-i tadwīr*)	48;26°/day (opposite Dirigent)	5) Small Sphere (*al-ṣaghīra*)	48;46°/day (cs)
5) Epicycle's Enclosing Orb (*muḥīṭ bi-tadwīr*)	24;23°/day (same direction as Dirigent)	6) Enclosing [Orb] (*al- muḥīṭa*)	24;23°/day (s)
6) Epicycle (*tadwīr*)	13;4°/day (cs)	7) Epicycle (*al-tadwīr*)	13;4°/day (cs)

Motion in the sequence (s) / counter-sequence (cs) of the signs is determined by the orb's apogee point

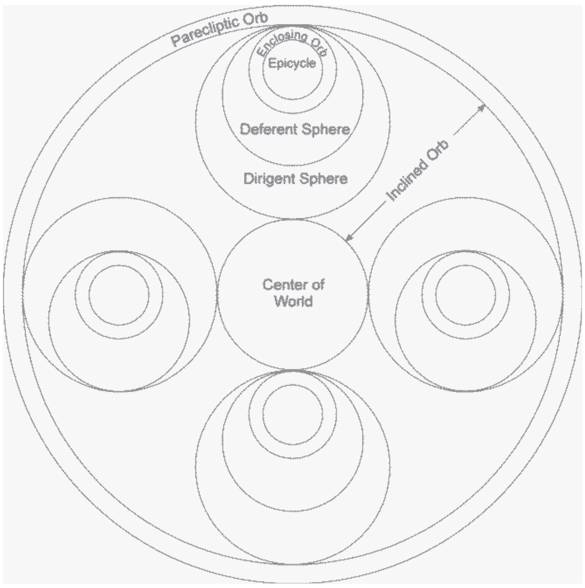

Fig. 3

Lunar Model from the Ḥall [6 orbs]

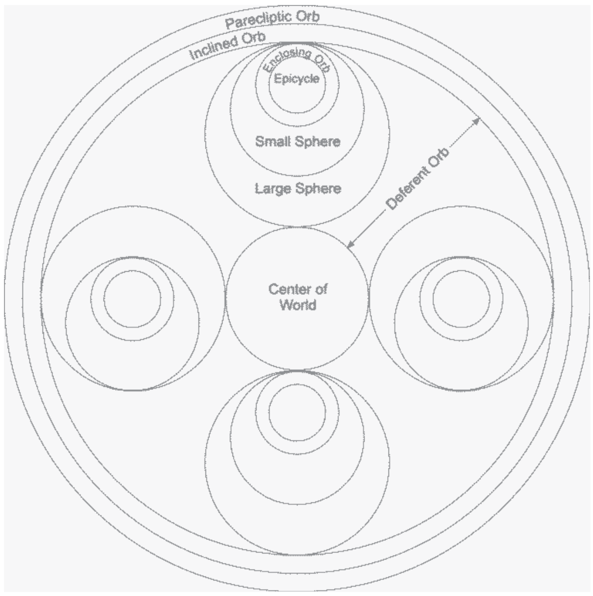

Fig. 4

Lunar Model from the Tadhkira [7 orbs]

In the *Tadhkira*, Ṭūsī has made a number of changes in the lunar model he first presented in the *Ḥall*. The most obvious is the change in terminology: “the dirigent orb” (*mudīr*) has now become the “large sphere” and the “epicycle’s deferent orb” (*ḥāmil*) has been renamed the “small sphere.” This is most likely due to the confusion resulting from using the terms dirigent and deferent, which are used for other parts of the planetary models, to designate the two outer spheres making up the Ṭūsī couple. Another more significant change is dividing the inclined orb of the *Ḥall* into two orbs in the *Tadhkira*, namely a different inclined orb (actually the inclined orb of the Ptolemaic model) and a different deferent. The resultant motion of these two orbs is 13;14°/day in the sequence of the signs, which is different from the 13;11°/day of the *Ḥall*’s inclined orb. Actually this corrects the mistake in the *Ḥall*, where Ṭūsī made the inclined orb move at the rate of the mean motion of the moon (*wasaṭ-i qamar*), apparently forgetting that this would result in the parecliptic motion being counted twice.

From this we can conclude that the rectilinear Ṭūsī couple and its applications to various planetary models emerged in stages and rather slowly. After coming up with the idea, apparently when writing the *Muʿīniyya*, it took many years before he felt comfortable enough presenting it in the *Ḥall.* But even then, the model still had a number of problems in both terminology and substance, which weren’t solved until the writing of the *Tadhkira* some 15 years later. But as we shall see, these differences will help us in tracing the transmission of the device and models. They will also help us make the case, almost a truism in the history of science, that such devices and models take time to evolve and be perfected. A sudden appearance of a complete and perfected theory or model should make us wary of claims of no transmission or influence.

Two Equal Circle Version = Plane Version with Equal Radii = The Pseudo-Curvilinear Version

In addition to the rectilinear version of the Ṭūsī couple, Naṣīr al-Dīn also developed a curvilinear version that was meant to produce a linear oscillation on a great circle arc. This was used to rectify a number of difficulties in Ptolemy’s latitude theory, as well as a curvilinear oscillation caused by the prosneusis point in the latter’s lunar model. In fact, as Ṭūsī mentions, it could be used wherever a curvilinear oscillation was needed, such as for motions of the celestial poles and vernal equinox if observation should showed such phenomena to be real.[11]

But before the final curvilinear version was introduced in the *Tadhkira* in 1261, it evolved slowly over a considerable period of Ṭūsī's lifetime. In the *Muʿīniyya*, when discussing the models for latitude, Ṭūsī notes that Ibn al-Haytham had dealt with this in a treatise and gives a brief sketch of his theory (Book II, Chap. 8). But he finds this solution lacking, and criticizes it without going into details since "this [work, i.e. the *Muʿīniyya*] is not the place to discuss it." Despite this criticism, Ṭūsī does not claim to have a solution for the problem of latitude, unlike the case with the longitudinal motions of the moon and planets.[12] In the *Ḥall*, Ṭūsī refrains from the earlier criticism of Ibn al-Haytham and instead presents the latter's model for latitude. Basically, this is an adaptation of the Eudoxan system of homocentric orbs, described in Aritstotle's *Metaphysics*, applied to Ptolemy's latitude models that used motion on small circles to produce latitudinal variation.[13] It is curious that Ṭūsī offers no model of his own, nor does he note, as he does later in the *Tadhkira*, that motions in circles will produce not only latitudinal variations but also unwanted longitudinal changes.

But a little over a year later, on 5 Shawwāl 644 / 13 February 1247 to be exact, Ṭūsī published a sketch of another version of his couple that was meant to resolve some of the difficulties of Ptolemy's latitude models.[14] This version was presented in the context of his discussion of these models in Book XIII of his *Taḥrīr al-Majisṭī* (Recension of the *Almagest*). After presenting a summary of Ptolemy's latitude model for the planets, and his special pleading regarding the complicated nature of these models that includes the endpoints of the epicycle

[11] See Ragep, *Naṣīr al-Dīn,* 1: 208-223, 2: 448-456.
[12] The relevant passages from Book II, Chaps. 5, 6 and 8 of the *Muʿīniyya*, with English translation, can be found in Ragep, "The Persian Context of the Ṭūsī Couple," pp. 113-130 on pp. 123-125.
[13] For details and an edition and translation of the relevant chapter from the *Ḥall*, see F. Jamil Ragep, "Ibn al-Haytham and Eudoxus: The Revival of Homocentric Modeling in Islam," in *Studies in the History of the Exact Sciences in Honour of David Pingree,* edited by Charles Burnett, Jan P. Hogendijk, Kim Plofker, and Michio Yano, pp. 786-809 (Leiden: E. J. Brill, 2004).
[14] This chronology contravenes G. Saliba's contention, followed by di Bono and others, that the "Two Equal Circle Version" in the *Taḥrīr* was the first occurrence of any version of the Ṭūsī couple. But clearly the dating of the *Ḥall* should put to rest this earlier proposal. Cf. George Saliba, "The Role of the *Almagest* Commentaries in Medieval Arabic Astronomy: A Preliminary Survey of Ṭūsī's Redaction of Ptolemy's *Almagest," Archives internationales d'histoire des sciences* 37 (1987): 3-20.

diameters rotating on small circles to produce latitude in a northerly or southerly direction,[15] Ṭūsī provides the following comment:

> I say: this discussion is outside the discipline (*ṣināʿa*) [201b] and is not persuasive for this matter. For it is necessary for a practitioner of this discipline to establish circles and bodies having uniform motions according to an order and arrangement [such that] from all of them [circles and bodies] these various perceived motions will be constituted. Then these motions being on the circumferences of the mentioned small circles, just as they result in the epicycle diameters departing from the planes of the eccentrics in latitude northward and southward, so too will they result in their departing from alignment with the center of the ecliptic, or of being parallel with diameters in the plane of the ecliptic with the exact same longitude, through accession and recession in the exact same amount of that latitude. And this is contrary to reality. And it is not possible to say that that difference is perceptible in latitude but not perceptible in longitude, since they are equal in size and distance from the center of the ecliptic.
>
> Now if the diameter of the small circle were made in the amount of the total latitude in either direction, and one imagines that its center moves on the circumference of another circle equal to it whose center is in the plane of the eccentric in the amount of half the motion of the endpoint of the diameter of the epicycle on the circumference of the first circle and opposite its direction, there will occur a shift to the north and south in the amount of the latitude without there occurring a forward or backward [motion] in longitude.
>
> In order to show this, let AB be a section of the eccentric and GD be from the latitude circle that passes through the endpoint of the diameter of the epicycle. And they intersect at E. EZ EM are the total latitude in the two directions. And EH is half of it in one of them. We draw about H with a distance EH a circle EZ and about E with a distance HE a circle HTKL. We imagine the endpoint of the diameter of the epicycle at point Z to move on circle EZ in direction G to B and the center H to move on circle HTKL in the direction

[15] This corresponds to the *Almagest*, Bk. XIII, Ch. 2.

G to A with half that motion. Then it is clear that when H traverses a quarter and reaches T, Z will traverse a half and reach E. Then when H traverses another quarter and reaches K, Z will traverse another half and reach M. And when H traverses a third quarter and reaches L, Z will traverse another half and will reach E once again. And when H completes a rotation, Z will return to its original place so that it will always oscillate in what is between ZM on the line GD without inclining from it in directions AB. This is the explanation of this method. However, it requires that the time the diameter is in the north is equal to the time it is in the south. However, in reality it is different than that. As for what is said regarding its motion on the circumference of a circle about a point that is not its center, as stated by Ptolemy, this needs consideration that can be verified according to what has preceded. We now return to the book.[16]

[16] *Taḥrīr al-Majisṭī*, Istanbul, Feyzullah MS 1360, ff. 201^{a} – 202^{a}

أقول هذا كلام خارج من الصناعة {٢٠١ب} غير مقنع في هذا الموضع فإنّ من الواجب على صاحب هذه الصناعة أن يضع دوائر وأجراماً ذوات حركات متشابهة على نضد وترتيب يتركّب من جميعها هذه الحركات المحسوسة المختلفة ثم إنّ كون هذه الحركات على محيط الدوائر الصغار المذكورة كما تقتضي خروج أقطار التداوير عن سطوح الخارجة المراكز في العرض شمالاً وجنوباً كذلك تقتضي خروجها عن محاذاة مركز البروج أو موازاة أقطار على سطح البروج بأعيانها في الطول إقبالاً وإدباراً بقدر تلك العروض بأعيانها وذلك مخالف للوجود ولا يمكن أن يقال أنّ ذلك التفاوت محسوس في العرض وغير محسوس في الطول لتساويهما في المقدار والبعد من مركز البروج فإن جعل قطر الدائرة الصغيرة بقدر جميع العرض في إحدى الجهتين وتوهّم أنّ مركزها يتحرّك على محيط دائرة أخرى مساوية لها مركزها في سطح الخارج المركز بقدر نصف حركة طرف قطر التدوير على محيط الدائرة الأولى وإلى خلاف جهتها حدث الانتقال إلى الشمال والجنوب بقدر العرض من غير أن يحدث في الطول تقدّم وتأخّر وليكن لبيانه اب قطعة من الخارج و جد من دائرة العرض المارة بطرف قطر التدوير وقد تقاطعا على ه و هز هم جميع العرض في الجهتين وهح نصفه في إحديهما ونرسم على ح ببعد هح دائرة هز وعلى ه ببعد حه دائرة ح ط ك ل ونتوهّم طرف قطر التدوير على نقطة ز متحرّكاً على دائرة هز في جهة جـ إلى ب ومركز ح متحرّكاً على دائرة ح ط ك ل في جهة جـ إلى ا نصف تلك الحركة فظاهر أنّه إذا قطع ح ربعاً وانتهى إلى ط قطع ز نصفاً وانتهى إلى ه ثم إذا قطع ح ربعاً آخر وانتهى إلى ك قطع ز نصفاً آخر وانتهى {٢٠٢آ} إلى م وإذا قطع ح ربعاً ثالثاً وانتهى إلى ل قطع ز نصفاً آخر وانتهى ثانياً إلى ه وإذا تم ح دورة عاد ز إلى موضعه الأول فهو دائماً يتردّد فيما بين زم على خطّ ج د غير مائل عنه إلى جهتي اب فهذا بيان هذا الوجه ولكن يلزم عليه أن يكون زمان كون القطر في الشمال مساوياً لزمان كونه في الجنوب والوجود بخلاف ذلك وأمّا القول بحركته على محيط دائرة حول نقطة غير مركزها على ما ذكر بطلميوس فمحتاج إلى نظر يحقّقه على ما مرّ ونعود إلى الكتاب

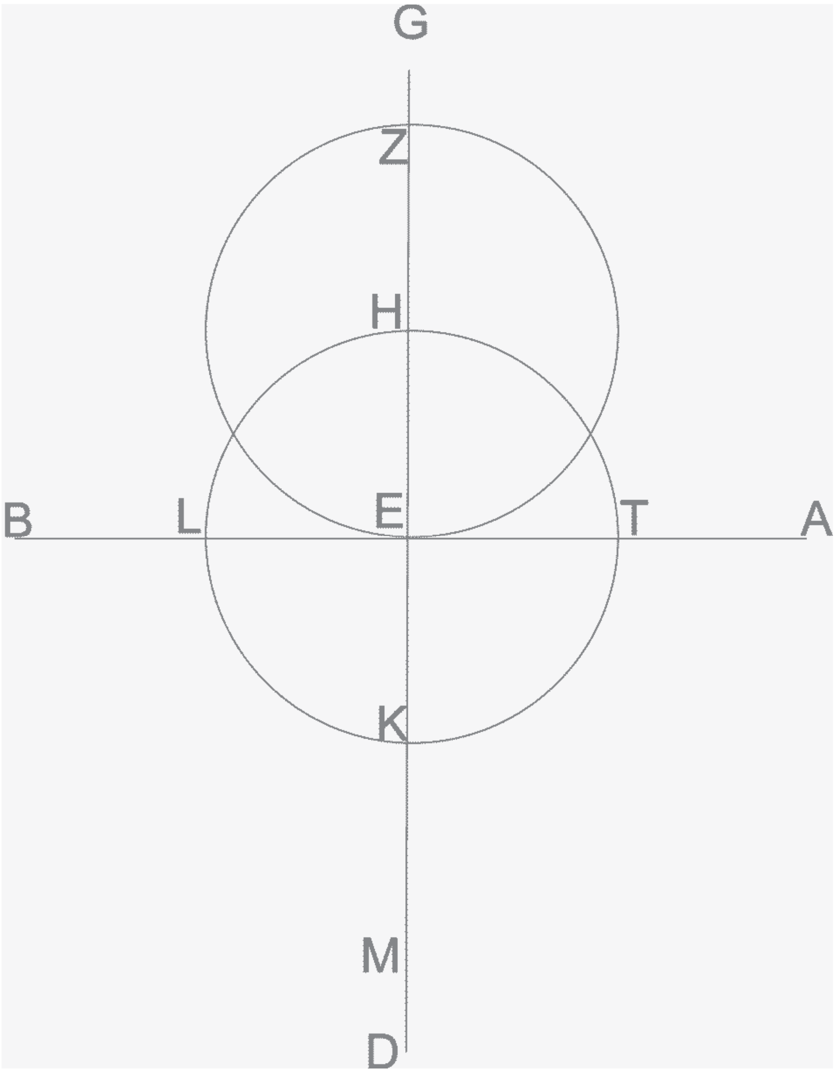

Fig. 5

The Two Equal Circle Version

There are several things we can say about this device. First of all, as Ṭūsī notes, it does not accurately model Ptolemy's latitude theory since it results in equal times in the north and in the south. Second, the motion of the epicycle endpoint is uniform with respect to the epicycle's mean apex, which again is contrary to what Ptolemy's model requires. Third, and more significant for our purposes, this model is actually a slightly modified version of the rectilinear Ṭūsī couple that was first presented in the *Ḥall.* The problem, though, is that the motion of the endpoint of the epicycle's diameter is on a straight line ZM, whereas the necessary motion should be on a great circle arc. This is curious. Surely Ṭūsī is aware that the motion in latitude should occur on the surface of a sphere; why, then, does he have this rather stripped-down version of his couple that can only result in rectilinear oscillation? The answer, it would seem, is that at this point he does not have a curvilinear version. He is dissatisfied with Ptolemy's small circles, and also realizes that Ibn al-Haytham's model does little more than provide a solid sphere basis for the inadequate small circles, but all he has to offer is a kind of vague notion that his couple might be modified to create the necessary motion in latitude. He clearly is still in the thinking stage.

The Complete Curvilinear Version = The Spherical Version with Oblique Axes and Equal Radii [Di Bono, p. 136]

Ṭūsī does not in fact offer a true curvilinear version until almost 15 years later (first part of Dhū al-qaʿda 659 / Sept.-Oct. 1261), at which time he publishes the first version of his *al-Tadhkira fī ʿīlm al-hayʾa*. In it, he puts forth a model consisting of three additional orbs enclosing the epicycle that are meant to produce a curvilinear oscillation that results in the motion in latitude (see Figures 6 and 7).[17] It is interesting that Ṭūsī presents this new model as a modification of Ibn al-Haytham's earlier attempt,[18] which, as we have seen, simply provides a physical basis for Ptolemy's small circles using homocentric orbs, which we may call the Eudoxan-couple (Figures 8 and 9). In addition to using the curvilinear version to resolve the difficulties related to the motion of the planetary epicycles in latitude, Ṭūsī notes it may also be used for moving the inclined orb of the two lower planets in latitude and for resolving the irregular motion brought about by the moon's so-called prosneusis point. Finally he states that this version could also be used to model the variable motion of precession ("trepidation") and the variability of the obliquity if these two motions were found to be real.[19] As we will see, these suggestions for extended usage of the couple turn out to be significant.

[17] Ragep, *Naṣīr al-Dīn,* 1: 216-221.
[18] For a fuller account of the curvilinear version, see Ibid., 2: 453-456. It should be noted that the curvilinear version does not in fact produce motion on a great circle arc; there is a small discrepancy resulting in a narrow, pinched figure-8 motion.
[19] Ibid., 1:220-223.

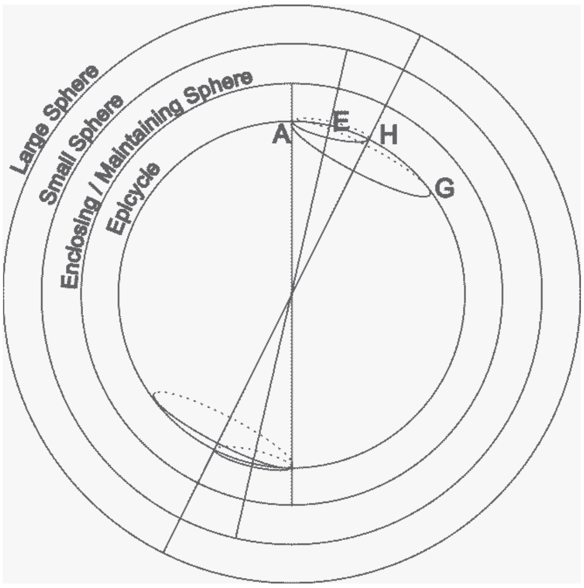

Fig. 6

Complete Curvilinear Version (3 spheres)

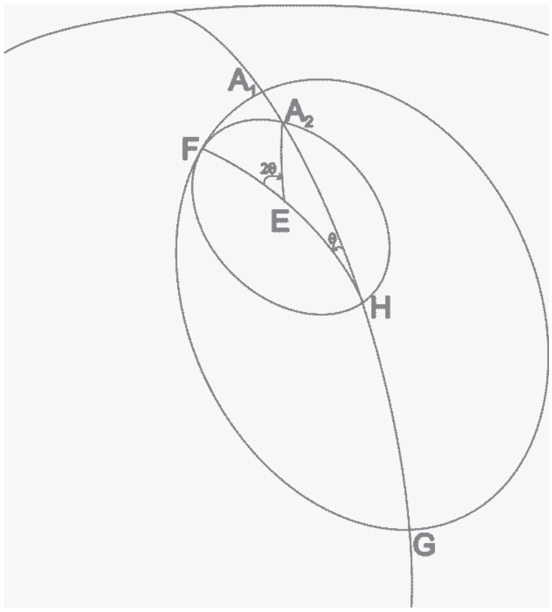

Fig. 7

Polar View showing motion of A (here endpoint of diameter of epicycle) along a great circle arc

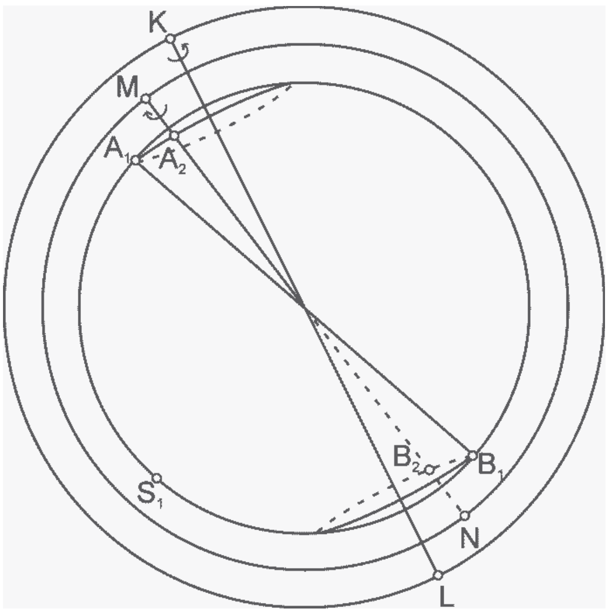

Fig. 8

Ibn al-Haytham's Eudoxan Couple (2 spheres)

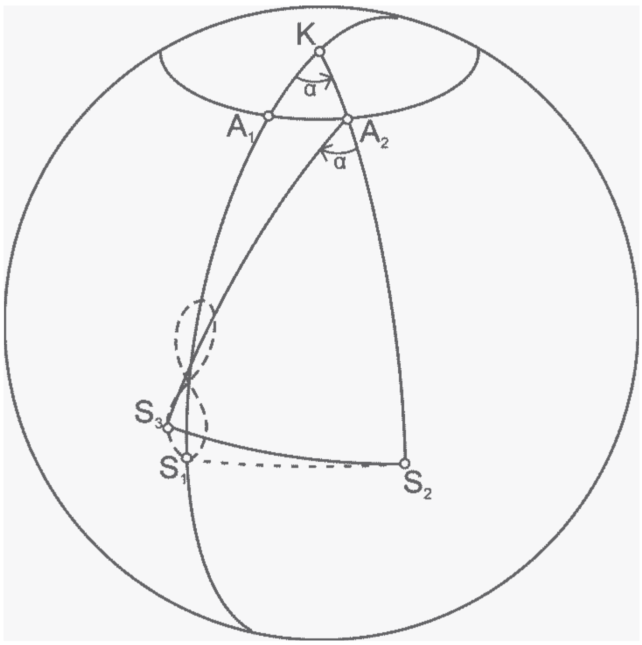

Fig. 9

Motion of A (endpoint of diameter of epicycle) on a circular path rather than a great circle arc

Use of the Couple for quies media

There is another issue related to the rectilinear couple that may have a bearing on tracing transmission. Quṭb al-Dīn al-Shīrāzī, one of Ṭūsī's associates in Marāgha and subsequently one of the eminent philosophers and scientists at Mongol courts in Tabrīz, remarks in his *al-Tuḥfa al-Shāhiyya*, written after Ṭūsī's death in 684/1285, that "it is possible to use this [lemma] to show the impossibility (*imtināʿ*) of rest between a rising and falling motion on the line (*samt*) of a terrestrial diameter."[20] The idea here is that the Ṭūsī couple, by showing that oscillating straight-line motion can be continuous, counters Aristotle's contention that there would be a "moment of rest" (*quies media*) between rising and falling.[21] This view was contested, and in fact Shams al-Dīn al-Khafrī (fl. 932/1525), in his comment on the *Tadhkira*, disputes Shīrāzī on this point. As we shall see, there are echoes in Latin Europe of this debate, which could well be due to transmission.

Sightings of the Ṭūsī-couple in non-Islamic Cultural Contexts before 1543[22]

We should note here that the development of the different versions of the Ṭūsī couple, and the models based upon them, took place over a 25-year period. The use, further development and discussion of the various versions of the couple in an Islamic context, such as we have noted above in the case of the *quies media* debate, can be traced over many centuries; the couple, which became known as the "large and small model" device (*aṣl al-kabīra wa-ʾl-ṣaghīra*), was incorporated into other theories and systems, as well as explained in a number of commentaries

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[20] Istanbul, Süleymaniye Library, Turhan H. Sultan MS 220, f. 34a:

ويمكن أن يجعل هذا دليلاً على امتناع السكون بين حركتين صاعدة وهابطة على سمت قطر من أقطار الأرض

[21] Tzvi Langerman, "Quies Media: A Lively Issue in Medieval Physics," in *International Ibn Sina Symposium Papers* (held in Istanbul, 22-24 May 2008), edited by Nevzat Bayhan, Mehmet Mazak, Nevzat Özkaya, and Raşit Küçük, 2 vols. (Istanbul: İstanbul Büyükşehir Belediyesi Kültür A.S. Yayınları, 2008 or 2009), 2: 53-67.

[22] The restriction of date will exclude a discussion of the translation into Sanskrit of part of ʿAbd al-ʿAlī al-Birjandī's (d. 1525-6) commentary on Tusi's *Tadhkira*, the part containing the presentation of the Ṭūsī couple. On this translation, see Takanori Kusuba and David E. Pingree, *Arabic Astronomy in Sanskrit: Al-Birjandī on Tadhkira II, Chapter 11, and Its Sanskrit Translation* (Leiden; Boston: Brill, 2002).

and independent works. There can be no question that these later developments and discussions in an Islamic context, in whatever language, can be traced back to one or more of Ṭūsī's works. However, when we cross cultural boundaries, the situation becomes less clear-cut, and here one is faced with a variety of opinions about the origin of "Ṭūsī-couple sightings" in these other cultural contexts. With the exception of one, and possibly a second, example, there are no cases of translations of Ṭūsī's writings on the Couple into non-Islamic languages. So in order to advocate that the appearance, or "sightings", of the Couple in other contexts is due to intercultural transmission, we will be faced in most cases with the need to postulate either non-extant texts or non-textual transmission. Such arguments will thus need to be based on plausibility rather than direct evidence; but on the other hand, many arguments of transmission in the history of science are based upon such plausibility arguments, and often become virtually irrefutable, especially when precise numeration is involved. The case for the transmission of the Ṭūsī-couple is not quite so iron-clad, but given the various types of evidence that can be brought to bear, I will argue that independent rediscovery, especially multiple times, becomes much less compelling.

But before presenting that evidence, let us list and discuss the various sightings. Because of the problematic nature of some of the material, especially in the case of Oresme, we will need to devote considerably more space to some examples than to others.

1) Transmission to Byzantium

The first know appearance of the Ṭūsī couple outside Islamic societies occurred around 1300, most likely through the efforts of a certain Gregory Chioniades of Constantinople, who is known for translating a number of astronomical treatises from Persian (or perhaps Arabic) into Greek.[23] Included in these works is a short theoretical treatise that has been dubbed "The

[23] These works occur in three codices, two in the Vatican and one in the Biblioteca Medicea Laurenziana in Florence.

Schemata of the Stars."[24] The lunar model in the Schemata uses the Ṭūsī couple, and there are diagrams in one of the codices that greatly resemble diagrams in Ṭūsī's works.[25]

As I argue in a recent paper, the *Schemata* is mostly a translation of certain parts of Ṭūsī's *Muʿīniyya*, with the Ṭūsī couple and lunar model coming from the *Ḥall-i Muʿīniyya*;[26] thus what we are dealing with is a case of the abridgement into Greek of a Persian original that we can confidently identify. It would seem that Chioniades was tutored by a certain Shams al-Dīn al-Bukhārī (almost certainly Shams al-Dīn Muḥammad ibn ʿAlī Khwāja al-Wābkanawī al-Munajjim), who used Ṭūsī's earlier Persian works rather than his revised and up-to-date *Tadhkira*.[27] Whether this was for linguistic reasons (Chioniades perhaps knowing Persian but not Arabic) or because of a reluctance to give a Byzantine access to cutting-edge astronomical knowledge, is unknown.[28] In any event, we can safely say that the version of the Ṭūsī couple and lunar model found in the *Schemata* came from the *Ḥall-i Muʿīniyya,* since both have 6 orbs for the lunar model and the same mistake in the inclined orb [13;11°/day (s) rather than the correct 13;14°/day (s)].[29]

The surprising conclusion is that the first known transmission of Ṭūsī's models came from his earlier Persian works, which contained a significant error. Furthermore, the only model transmitted was the lunar model, and there is no hint in the *Schemata* of the models for latitude, either from the *Taḥrīr* or from the *Tadhkira*. Nevertheless, there can be no question that some of Ṭūsī's innovations had made their way into Greek by the early 14th century, and the existence in

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[24] Edition and translation in E. A. Paschos, and P. Sotiroudis, *The Schemata of the Stars: Byzantine Astronomy from A.D. 1300* (Singapore; River Edge, NJ: World Scientific, 1998), pp. 26-53.
[25] This was first recognized by Otto Neugebauer, who reproduced diagrams from Vatican Gr. 211, f. 116r in his A History of Ancient Mathematical Astronomy, 3 vols. (New York: Springer-Verlag, 1975), 3: 1456.
[26] F. Jamil Ragep, "New Light on Shams: The Islamic Side of Σὰμψ Πουχάρης," in *Politics, Patronage, and the Transmission of Knowledge in 13th - 15th Century Tabriz*, edited by Judith Pfeiffer, pp. 231-247 (Leiden: E. J. Brill, 2014).
[27] This can most easily be established from the list of star names in the *Schemata*, pp. 30-37; see Ragep, "New Light on Shams," pp. 239, 241-242.
[28] It was reported that there was great reluctance by the Persians to teach astronomy to a Byzantine because of a legend that doing so would lead to the former's demise; see Ragep, "New Light on Shams," pp. 231-232.
[29] *Schemata,* pp. 42-45; for the *Ḥall*, see above and "New Light on Shams," p. 242.

Italy of the only three known manuscript witnesses strongly suggests that the transmission of this knowledge had made it into the Latin world by the 15th century.[30]

We should also mention here that since Chioniades read the *Ḥall-i Muʿīniyya,* he would no doubt have been exposed to Ibn al-Haytham's latitude theory, which made up Chapter 5 of that work.[31] This may well have relevance to the question of how that rather obscure theory might have reached scholars in Latin Europe.

2) The Ṭūsī and Eudoxan Couples in Latin Europe

Historians have identified multiple sightings of the Ṭūsī and Eudoxan (i.e., Ibn al-Haytham's) couples in Latin Europe, starting in the fourteenth century. Here is a chronological list that is certainly not exhaustive:

a) Avner de Burgos

The Jewish philosopher and polemicist Avner de Burgos (ca. 1270-1340), who as a convert to Christianity was know as Alfonso de Valladolid, proved a theorem identical to a rectilinear Ṭūsī Couple. Tzvi Langermann has noted that Avner/Alfonso "adduces his theorem in a mathematical context, the stated purpose of which is 'to construct (*li-ṣayyer*) a continuous and unending rectilinear motion, back and forth along a finite straight line, without resting when reversing direction [literally: "between going and returning"]'."[32] What is interesting here is that this use of the couple, as part of the *quies media* debate, is not something one finds in Ṭūsī but is

[30] Pingree states that Vatican gr. 211 is listed in the Vatican inventory of 1475, while Vatican gr. 1058 is listed in the inventory of ca. 1510 but may well have been in the collection earlier: David Pingree, *The Astronomical Works of Gregory Chioniades*, vol. 1: The *Zīj al-ʿAlāʾī* (Amsterdam: J. C. Grieben, 1985), pp. 23, 25.
[31] F. Jamil Ragep, "Ibn al-Haytham and Eudoxus: The Revival of Homocentric Modeling in Islam," in Charles Burnett, Jan P. Hogendijk, Kim Plofker, and Michio Yano (eds.), *Studies in the History of the Exact Sciences in Honour of David Pingree* (Leiden: E. J. Brill, 2004), pp. 786-809.
[32] Y. Tzvi Langermann, "Medieval Hebrew Texts on the Quadrature of the Lune," *Historia Mathematica* 23, no. 1 (1996): 31-53, on p. 34.

to be found in the work of his associate and student Quṭb al-Dīn al-Shīrāzī. As we will see, this may well have implications for the transmission of the couple to Europe.

b) Nicole Oresme

Nicole Oresme (ca. 1320-1382) in his *Questiones* on Sacrobosco's *De spera* describes some sort of model that will produce reciprocating rectilinear motion from three circular motions. Now both Garrett Droppers and Claudia Kren raised the possibility that Oresme had somehow been influenced by "Ṭūsī's device".[33] Recently, André Goddu has challenged this and has raised another possibility, namely that Oresme hit upon a solution similar to Ṭūsī's for producing rectilinear motion from straight-line motion (though still leaving open the (weak?) alternative that Oresme may have come across some description of it).[34] Because Goddu's speculations, which we will discuss below, depend upon several misinterpretations of both Ṭūsī and Oresme, we will need to carefully consider what Oresme is proposing. Here is a slightly revised version of Kren's translation of the relevant passage:[35]

> Concerning this problem [i.e., whether celestial bodies move in circular motion], I propose three interesting conclusions. First, it is possible for some planet to be moved perpetually according to its own nature in a rectilinear motion composed of several circular motions. This motion can be brought about by several intelligences, any one of which may endeavor to move in a circular motion, *nor would this purpose be in vain* [rev: and is not frustrated in this endeavor].
>
> Proof: Let us propose, conceptually, as do the astrologers, that A is the *deferent* [rev: deferent circle] of some planet, or its center; B is the *epicycle* [rev: epicycle circle] of the

[33] Garrett Droppers, "The Questiones de Spera of Nicole Oresme: Latin Text with English Translation, Commentary and Variants," Ph.D. dissertation, University of Wisconsin, 1966, pp. 462-464; Claudia Kren, "The Rolling Device of Naṣīr al-Dīn al-Ṭūsī in the *De spera* of Nicole Oresme?" *Isis* 62 (1971): 490-498.
[34] André Goddu, *Copernicus and the Aristotelian Tradition: Education, Reading, and Philosophy in Copernicus's Path to Heliocentrism* (Leiden [The Netherlands]; Boston: Brill, 2010), pp. 481, 484.
[35] The parts of Kren's translation (p. 490) that have been changed are in italics; my suggested revisions (rev.) are in brackets immediately following. Droppers also provides a translation (pp. 285, 287, 289), somewhat more literal than Kren's, that I have also taken into account.

same planet; and C is the body of the planet, or its center; I take these as equivalent [the same?]. Let us also imagine line BC from the center of the epicycle to the center of the planet, and CD, a line in the planet on which BC falls perpendicularly. Let circle A move on its center toward the east, and B toward the west. The planet, C, revolves on its own center toward the east. Moreover, since line BC is of constant length, because it is a radius, let us propose that the *distance* [rev: amount] B descends *in* [rev: according to] the motion of the deferent is *the distance which* [rev: as much as] point C *may ascend* [rev: ascends] with the motion of the epicycle. From this one can obviously observe that point C in some definite time will be moved in a straight line. *Let us then further assume that point B would ascend by its own motion on just the circumference on which it may descend with the motion of the planet.* [rev: Let us then further assume that the circuit on which B would ascend by its own motion is as much as the motion of the planet descends.] It is further clear that point D will move continually on the same line; thus the entire body of the planet will be moved to some terminus in a rectilinear motion and will return again with a similar motion.[36]

In order to analyze this passage, and to understand Oresme's intention, we should note from the last sentence that the *body* of the planet is meant to move rectilinearly. Furthermore, not

[36] Here is Kren's Latin version (p. 491, n. 3); cf. Droppers, pp. 284, 286, 288:

Circa hanc questionem, pono 3 pulcras conclusiones. Prima est quod possibile est quod aliquis planeta secundum quodlibet sui moveatur in perpetuum motu recto composito ex pluribus motibus circularibus, ita quod iste motus proveniat a pluribus intelligentiis quarum quelibet intenderet movere motu circulari nec frustratur ab intentione sua.

Pro cuius probatione, suponatur per ymaginationem, sicut faciunt astrologi, quod A sit circulus deferens alicuius planete, vel centrum eius, et sit B circulus epiciclus eiusdem planete, et C sit corpus planete vel centrum eius; hoc habeo pro eodem. Et ymaginetur linea BC, exiens de centro epicicli ad centrum planete, et CD sit linea in planeta supra quam alia cadat perpendiculariter. Moveatur etiam A circulus supra centrum ad orientem, et B ad occidentem, et C planeta supra centrum suum volvatur ad orientem. Cum ergo linea BC semper sit equalis, quia est semidyameter, ponatur quod quantum B descendit ad motum deferentis, tantum C punctus ascendat per motum epicicli. Ex quo patet intuenti quod punctus C per aliquod certum tempus movebitur super lineam rectam. Tunc ponatur ultra quod perifora qua punctus B ascenderet motu suo tantum descendat motu planete. Et patet iterum quod punctus D continue movebit in eadem linea. Ergo totum corpus planete movebitur motu recto usque ad aliquem terminum, et iterum poterit reverti in motu consimilli.

only does the center of the planet C move in a straight-line but a certain point D, which is the end-point of a planetary radius CD, does as well.

Now Droppers, and Goddu who follows him, do not take the rectilinear motion of D into account; inexplicably, both have D at the end of a planetary radius whose starting point is C, the center of the planet (see Fig. 10).[37]

[37] Figure is from Droppers, *Questiones de Spera*, p. 287; reproduced by Goddu, *Copernicus,* p. 481. Note that despite the use of *corpus* in referring to the planet, Goddu insists that "there is no indication that Oresme was directly concerned with the physical characteristics of the bodies or the mechanisms" (p.481). This may be why both Droppers and Goddu seem capable of ignoring Oresme's clear statement that it is the "entire body of the planet" that moves in a straight line. We should also note here that the title of this *Questio* is "Whether any heavenly body (*corpus celeste*) is moved circularly."

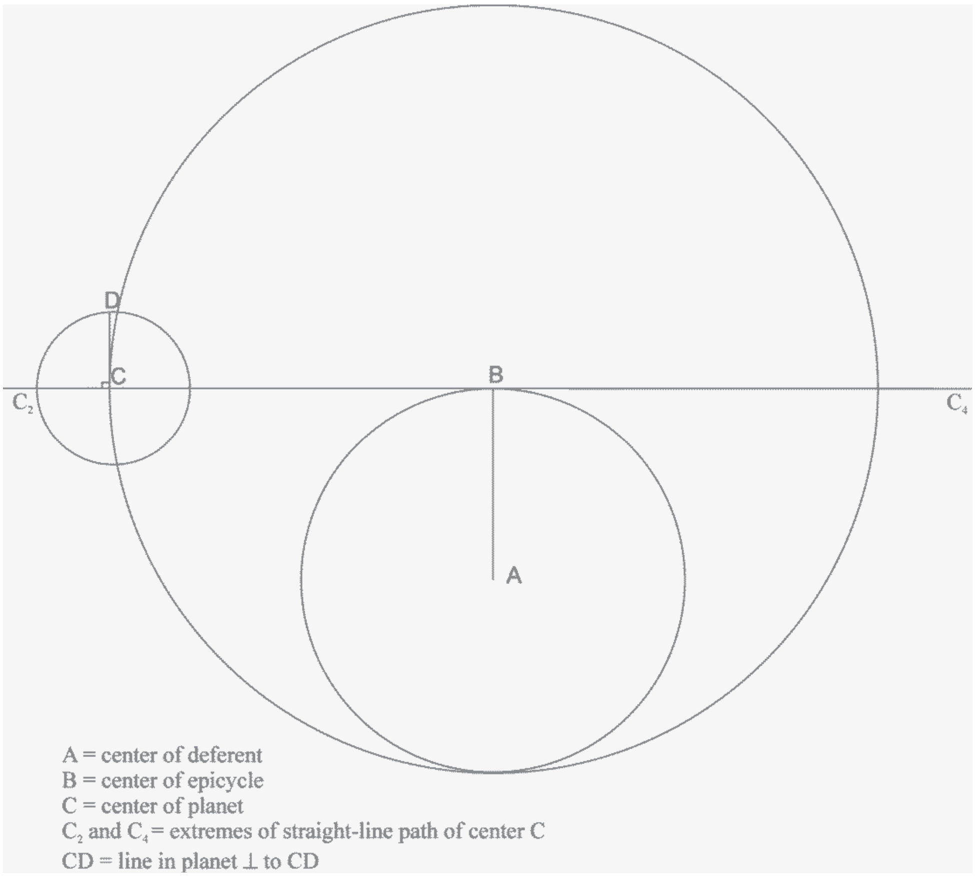

Fig. 10

Oresme's Construction as Proposed by Droppers

Kren, on the other hand, does follow Oresme's text and provides a plausible reconstruction based upon a more or less correct interpretation of Ṭūsī's *Tadhkira* as she found it in Carra de Vaux's flawed 1893 French translation. Oresme provides no diagram, and Kren must admit that "as it appears in Oresme's *Questiones de spera*, the passage makes no sense whatsoever."[38] Nevertheless, following Kren's lead and making a few modifications, I believe we can reconstruct both Oresme's model and his intention.[39] In essence, what Kren proposes is that Oresme is not discussing the simple 2-circle Ṭūsi-couple that results in the rectilinear

[38] Kren, "Rolling Device," p. 492.

[39] Goddu, on the other hand, finds Kren's reconstruction "implausible" (*Copernicus*, p. 480), but this seems to be based on the grounds that Ṭūsī's construction requires 2 circles while Oresme's requires 3. He apparently is unaware of Ṭūsī's physicalization of his geometrical device and his explicit use of 3 spheres in the *Tadhkira*; see Ragep, *Naṣīr al-Dīn,* 1: 200-201, 350-351 and 2: 435-437. Again, Kren is explicitly depending on an earlier French translation of this passage in which Ṭūsī describes how to physicalize his device (Kren, "Rolling Device," p. 493, fn. 8.), so Goddu's claim that Ṭūsī does not have a 3-sphere model is odd, to say the least.

oscillation of a point between two extrema but rather Ṭūsi's physicalized version, which we have already encountered in Fig. 2.[40]

Let us take A to be the center of the deferent, B the center of the epicycle, and C the center of the planet. The solid lines indicate the outer surfaces of solid bodies, while the dotted lines indicate "inner equators" of these solid bodies. Note that the solid orbs are the actual moving bodies; they "accidentally" produce the mathematical Ṭūsī couple indicated by the broken lines. Now for this model to work, the epicycle B needs to move with twice the angular speed as deferent A and in the opposite direction. This will then result in the planet's center C oscillating on a straight line. This will not, however, result in the apex of the planet, D, moving rectilinearly. As shown in the diagram, when the deferent and epicycle have rotated from an initial position (where A, B, C, and D were on the same line), D will move from D_0 to D_1. In order to deal with this issue, Ṭūsī introduces what he calls an enclosing sphere (*kura muḥiṭa*), which is shown in the diagram as an orb enclosing and concentric with the planet C. This orb would then have the job of moving D from D_1 back to its initial position D_0. Since $\angle$ BAC = $\angle D_0CD_1$, the enclosing sphere needs to move with the same speed and direction of the deferent in order to keep D oscillating on the straight line.

Kren has assumed that Oresme is simply copying Ṭūsī's Physicalized Rectilinear Version, and she has some tortured readings that would introduce this fourth, enclosing orb into Oresme's account. But Oresme clearly says he only needs three, and in fact Ṭūsī's commentators indicate that one could replace orb C and the enclosing orb by combining their motions into a single orb. Now Ṭūsī does not do this, probably because for him orb C is an epicycle, not an otherwise stationary planet, and he does not want to lose its parameters, which are critical for Ptolemaic planetary theory, by combining it with another orb. But Oresme has no such constraints since for him the construction does not represent an actual planetary model. So the planet C can move as needed, in this case with just the rotational direction and speed of deferent A that will keep line CD aligned with the line of oscillation.

[40] This is a modified version of what is described in the *Tadhkira*, II.11[4] (Ragep, *Naṣīr al-Dīn* pp. 200-201); cf. Fig. C13 (p. 351). For a discussion of this passage, see ibid., pp. 435-438.

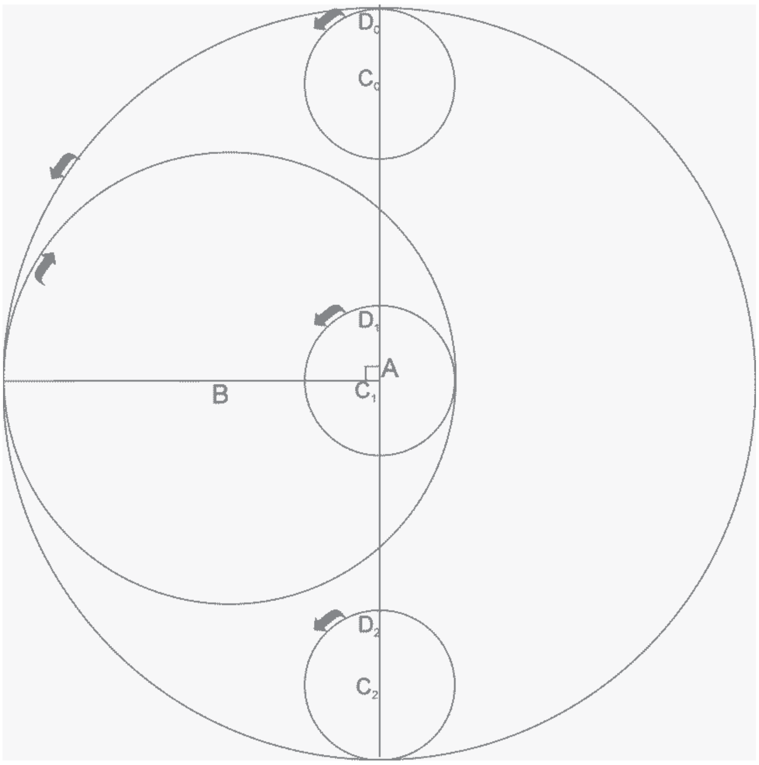

Fig. 11

Oresme's Physicalized Rectilinear Version of the Ṭūsī Couple

How well does this interpretation fit with the existing text? Actually rather well, all things considered. Turning to Fig. 11, let us go through the various features as presented by Oresme:

a) A is the deferent, which "carries" (*deferre*) the epicycle B; the planet C is moved by the epicycle. According to most standard medieval accounts, and presumably this is what Oresme intends by referring to the conceptualization of the astrologers, the epicycle is embedded in the deferent and the planet is embedded in the epicycle as shown.

b) A radius CD of the planet would in general not be perpendicular to the line BC in this construction; however, as indicated, it would be at the quadratures as noted by Kren. As mentioned above, the alternative given by Droppers and followed by Goddu (Fig. 10) does not fit the stipulation that D remain on the line of oscillation.

c) The directions of the motions (A eastward, B westward, C eastward) is consistent with Ṭūsī's model.

d) Oresme emphasizes that BC is a radius of constant length, which probably indicates that he is aware that this is part of the proof for the Ṭūsī couple. For this to work so that point C will remain on a straight line, Oresme would need to make B rotate twice as fast as A (or in his terms point B would descend due to A, while C would ascend with twice the speed due to B). However, he seems to imply that the deferent and epicycle rotate at the same speed (or descend and ascend in equal amounts). Unless he has some other sense for ascend and descend, Oresme does not seem to be in control of this rather critical part of the model.

e) If one accepts my emended translation, then Oresme does understand that the planet will need to rotate in the direction opposite that of the epicycle. Again we are not provided with any amounts, but it seems that Oresme is conceiving of D_0 being displaced to D_1 by the "ascending" motion of B, which would then need to be countered by the descending motion of the planet (see Fig. 2). The flow of the argument is then clear: he starts with the "proof" that C will oscillate on a straight line and then follows with his "proof" that D will follow suit through the additional motion of the planet.

What conclusions can we reach? Evidently Oresme is aware of what we may call Naṣīr al-Dīn's physicalized Ṭūsī-couple as presented in the *Tadhkira*. But Oresme makes no claim to have invented this on his own; and his presentation is so garbled and mistake-ridden that it would be implausible in the extreme to assume that he re-invented this model. On the other hand, the 3-sphere version Oresme presents, as a deferent-epicycle-planet construction, is not to be found explicitly in Ṭūsī or other Islamic sources I am aware of; thus it seems likely that Oresme or an intermediary had adapted the model for this philosophical discourse. Finally, we should note that there is an echo of the use of the Ṭūsī-couple for the *quies media* debate that we first encountered with Quṭb al-Dīn al-Shīrāzī. Oresme states that: "By the imagination, it is possible that rectilinear motion be eternal, with the exception that in the point of reflection the movable would not be said to be moved nor at rest."[41]

[41] Droppers, *Questiones de Spera,* p. 291.

c) Ibn Naḥmias

In his *Light of the World*, Joseph ibn Joseph ibn Naḥmias, a Spanish Jew living in Toledo ca. 1400, used a double circle device in his astronomical models that is virtually identical to Ṭūsī's from *Taḥrīr al-Majisṭi*, which I have referred to above as the "two equal circle version" and the pseudo-curvilinear version. He also incorporates it in his recension of *Light of the World.* Note that despite living in the Christian part of the Iberian penisula, Ibn Naḥmias wrote *Light of the World* in Judeo-Arabic (Arabic in Hebrew script), though the recension is in Hebrew. Robert Morrison in another essay in this volume has detailed Ibn Naḥmias's use of the Ṭūsī couple and has also discussed the vexed question of its possible transmission to Ibn Naḥmias and other Jewish scholars.[42] We shall return to this question below.

d) Georg Peurbach

From an extensive mathematical analysis of the 1510 and 1512 annual ephemerides of Johannes Angelus, Jerzy Dobrzycki and Richard Kremer concluded that they were based upon modifications of the *Alfonsine Tables,* these modifications consisting of mechanisms meant to produce harmonic motion that were somehow added to the standard Ptolemaic models.[43] Because Angelus seems to indicate that these were based upon a new table of planetary equations due to Georg Peurbach (d. 1461), Dobrzycki and Kremer speculate that the underlying models used by Peurbach incorporated one of the "Marāgha" models, perhaps the Ṭūsī-couple or the mathematically equivalent epicycle/epicyclet of Ibn al-Shāṭir. Aiton has also raised the possibility that Peurbach in his *Theoricae novae planetarum* may be referring to Ibn al-Haytham's Eudoxan-couple when he states that "On account of these inclinations and slants of the epicycles, some assume that small orbs have the epicycles within them, and that the same things happen to their motion."[44] Although speculative, these authors do point to the possibility that European astronomers in the late 15th and early 16th centuries, other than Copernicus, used

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[42] See R. Morrison, "Jews as Scientific Intermediaries in the European Renaissance".

[43] J. Dobrzycki and R. L. Kremer, "Peurbach and Marāgha Astronomy? The Ephemerides of Johannes Angelus and Their Implications," *Journal for the History of Astronomy* 27 (1996): 187-237.

[44] E. J. Aiton, "Peurbach's *Theoricae novae planetarum:* A translation with Commentary," *Osiris,* 2nd ser., 3 (1987): 5-43, on p. 36 and n. 118.

and adapted devices that we normally associate with Islamic astronomy. This is an important point that we will revisit when we discuss some of the objections that have been raised to astronomical transmission from Islam to Latin Europe.

e) Johann Werner

In his *De motu octavae sphaerae*, Werner (1468-1522) apparently used the Two Equal Circle Version to generate linear harmonic motion. According to Dobrzycki and Kremer,

> "Werner allotted the trepidational motion of "Thabit's" and Peurbach's models to the solstitial points of two concentric spheres. Two circles of trepidation, of equal radii and centred on the solstitial points of the next higher sphere, rotate in opposite directions so that trepidational variations in longitude do not introduce shifts in the obliquity of the ecliptic. Werner thus managed to generate linear harmonic motion by the uniform motions of two circles."[45]

f) Giovanni Amico

Giovanni Battista Amico (d. 1538) used the "3-sphere curvilinear version" as described in the *Tadhkira* in his *De motibus corporum coelestium*;[46] in other words, he used the version with three spheres, two for producing the curvilinear oscillation on the surface of a sphere and the third functioning as a counteracting sphere so that only the curvilinear oscillation of its pole

[45] Dobrzycki and Kremer, "Peurbach and Marāgha," p. 233, fn. 53. Much more attention has been paid to Copernicus's "Letter against Werner" than to Werner's actual treatise, which needs more extensive investigation. The following is some of the secondary literature: Jerzy Dobrzycki, "Astronomical Aspects of the Calendar Reform," in *Gregorian Reform of the Calendar,* ed. by G. V. Coyne, M.A. Hoskin, and O. Pedersen (Vatican City, 1983), 117-26, on p. 122; Jerzy Dobrzycki, "Teoria precesji w astronomii średniowiecznej," *Studia i materiały z dziejów nauki polskiej,* ser. c, xi (1965), 3-47, on pp. 29-32; and Anne Räumer, "Johannes Werners Abhandlung "Über die Bewegung der achten Sphäre" (De motu octavae sphaerae, Nürnberg 1522)," *Wolfenbütteler Renaissance Mitteilungen* 12(1988): 49-61.

[46] On Amico, see Noel Swerdlow, "Aristotelian Planetary Theory in the Renaissance: Giovanni Battista Amico's Homocentric Spheres," *Journal for the History of Astronomy,* 3 (1972): 36-48 and Mario di Bono, *Le sfere omocentriche di Giovan Battista Amico nell'astronomia del Cinquecento. Con il testo del "De motibus corporum coelestium ...*" (Genoa, 1990).

is transmitted to the next lower sphere.[47] According to di Bono, "It is of particular interest that in the 1537 [revised] edition of his work Amico is aware that on the surface of a sphere the demonstration does not function as it should; but since the inclination of the axes is not great, he considers the error negligible."[48]

g) Fracastoro

Girolamo Fracastoro in his *Homocentrica*, published in 1538, referred to a device for producing rectilinear motion, but does not incorporate it into his astronomy. The description and diagram makes it clear that he is referring to the "Two Equal Circle Version".[49]

h) Copernicus

Swerdlow and Neugebauer succinctly summarize Copernicus's use of the various devices invented by Ṭūsī:[50]

> "In *De revolutionibus* he uses the form of Ṭūsī's device with inclined axes for the inequality of the precession and the variation of the obliquity of the ecliptic, and in both the *Commentariolus* and *De revolutionibus* he uses it for the oscillation of the orbital planes in the latitude theory. In the *Commentariolus* he uses the form with parallel axes for the variation of the radius of Mercury's orbit, and by implication does the same in *De revolutionibus* although without giving a description of the mechanism."

However, we will need to examine the situation a bit more closely.[51] Let us take *De revolutionibus* first. In fact, the device put forth and the proof given in III.4 for variable

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[47] Ṭūsī refers to this third as "the enclosing sphere" (*al-kura al-muḥīta*) (Tadhkira, pp. 220-221); Amico calls it a "withstanding" (*obsistens*) sphere" (Swerdlow, "Aristotelian Planetary Theory," p. 41.).
[48] Di Bono, "Copernicus, Amico, Fracastoro and Ṭūsī's Device," p. 141; Ṭūsī does not mention this problem, but it is mentioned by at least one commentator on the *Tadhkira*; see Ragep, *Tadhkira*, p. 455.
[49] Di Bono, "Copernicus, Amico, Fracastoro and Ṭūsī's Device," pp. 143-144.
[50] N. M. Swerdlow and O. Neugebauer, *Mathematical Astronomy in Copernicus's De Revolutionibus* (New York: Springer-Verlag, 1984), p. 47.
[51] Here we follow Di Bono's lead in his "Copernicus, Amico, Fracastoro and Ṭūsī's Device," esp. pp. 138-141.

precession and the variation of the obliquity are, *pace* Swerdlow and Neugebauer, for the "Two Equal Circle Version," not for the 2- or 3-sphere curvilinear version (i.e. "Ṭūsī's device with inclined axes"). And in all other cases in which he uses it in *De Revolutionibus* (for Mercury's longitude model in V.25 and for the latitude theory in VI.2), Copernicus refers the reader back to III.4. We may then conclude that Copernicus wishes to use the "Two Equal Circle Version" exclusively in *De revolutionibus*. As Swerdlow and Neugebauer note, Copernicus's statement that he will be using chords rather than arcs (as necessitated by the use of the rectilinear rather than curvilinear version) is reasonable since the deviation from a curvilinear version is relatively minor.[52] But it does raise questions about the kind of modeling Copernicus uses in *De revolutionibus* in contrast to the *Commentariolus*. For in the *Commentariolus,* it is the truncated two-sphere curvilinear version that is used for the latitude models[53] and it is the Physicalized Rectilinear Version that is used to vary the radius of Mercury's orbit, but in a truncated, 2-sphere version without the enclosing/maintaining sphere.[54] The conclusion would seem that Copernicus was attempting to provide actual spherical models for the two versions of the Ṭūsī couple he uses in the *Commentariolus*, but that he cut a corner or two by not dealing with the disruption of the contained orb, which, after all, is why Ṭūsī (and Amico) have their maintaining (or withstanding) spheres. In *De revolutionibus*, Copernicus abandons any pretense of full physical models for his Ṭūsī couples and instead just relies on the Two Equal Circle Version, which, as we have seen, is a mathematical, not a physical, model.[55]

The Transmission Skeptics

Although difficult to gauge in a precise way, impressionistically it would seem that a majority of historians of early astronomy have accepted, to a lesser or greater degree, the influence of late Islamic astronomy on early modern astronomers and in particular on Copernicus. This was perhaps most explicitly set forth by Swerdlow and Neugebauer: "The

[52] Swerdlow and Neugebauer, *Mathematical Astronomy,* p.136.
[53] N. M. Swerdlow, "The Derivation and First Draft of Copernicus's Planetary Theory: A Translation of the Commentariolus with Commentary," *Proceedings of the American Philosophical Society* 117 (1973): 423-512, on pp. 483, 497.
[54] Ibid., p. 503.
[55] Cf. Di Bono, "Copernicus, Amico, Fracastoro and Ṭūsī's Device," pp. 140-141.

question therefore is not whether, but when, where, and in what form he [Copernicus] learned of Marāgha theory."[56]

Nevertheless, there have been a number of skeptics who have raised various issues that are worth exploring. In 1973, for example, I. N. Veselovsky called attention to what is the converse of the Ṭūsī-couple, namely a device for producing a circular motion from straight-line motions, which was set forth by Proclus in his commentary on the First Book of Euclid's *Elements*.[57] Copernicus refers to just this passage in Proclus when he uses the Ṭūsī-couple for his Mercury model.[58] But there are numerous problems with attributing Copernicus's source to Proclus rather than Ṭūsī. In the first place, Proclus, as mentioned, is setting forth a way to produce circular motion from linear motions, which is the opposite of what the Ṭūsī couple does. Second, it has been well established by Swerdlow and E. Rosen that Copernicus only received a copy of Proclus's book in 1539 as a gift from Rheticus, which is many years after first using the couple in the *Commentariolus*.[59] Di Bono proposes, as a way to save Veselovsky's suggestion, the possibility that Copernicus may have seen a copy of the original Greek while in Italy, this gaining some plausibility because it was part of the library that Cardinal Bessarian had bequeathed to the Venetian Senate.[60] But again this raises numerous other problems, namely that Copernicus is then required to read, or have read to him, a Greek manuscript, that he was inspired by an obscure passage talking about something only vaguely related to a device that, as

[56] Swerdlow and Neugebauer, *Mathematical Astronomy,* p. 47. The emphatic way in which this is stated is most likely due more to Swerdlow than Neugebauer, for cf. the latter's earlier remark that "The mathematical logic of these methods is such that the purely historical problem of contact or transmission, as opposed to independent discovery, becomes a rather minor one" (O. Neugebauer, "On the Planetary Theory of Copernicus", *Vistas in astronomy* 10 (1968): 89–103, on p. 90; reprinted in *idem*, *Astronomy and History* (New York, 1983), 491–505, on p. 492). E. S. Kennedy and Willie Hartner also entertained little doubt that Copernicus's work was heavily influenced by his Islamic predecessors (E. S. Kennedy, "Late Medieval Planetary Theory", *Isis*, 57 (1966): 84-97 and Willy Hartner, "Copernicus, the Man, the Work, and its History," *Proceedings of the American Philosophical Society* 117 (1973): 413-422). A recent rejoinder to Goddu's skepticism regarding an Islamic influence on Copernicus has been made by Peter Barker and Matjaž Vesel, "Goddu's Copernicus: An Essay Review of André Goddu's *Copernicus and the Aristotelian Tradition*," *Aestimatio* 9 (2012): 304-336, esp. 327-332. Goddu's answer, in which he distances himself from an outright rejection of Islamic influence, can be found in "A Response to Peter Barker and Matjaž Vesel, 'Goddu's Copernicus'", *Aestimatio* 10 (2013): 248-276, esp. 251-254.
[57] Ivan Nikolaevich Veselovsky, "Copernicus and Naṣīr al-Dīn al-Ṭūsī," *Journal for the History of Astronomy* 4 (1973): 128-130.
[58] *De revolutionibus*, Bk. V, ch. 25.
[59] See Ragep, Naṣīr al-Dīn, 2: 430-432 for an elaboration.
[60] Di Bono, "Copernicus, Amico, Fracastoro and Ṭūsī's Device," p. 146.

we have seen, was certainly available from other sources. And Copernicus himself doesn't even get the reference to Proclus correct; he has Proclus claiming that "a straight line can also be produced by multiple motions,"[61] but as we have seen Proclus refers to the production of a circle, not a straight line. And in any event, Copernicus himself mentions "some people" who refer to the Ṭūsī device as producing "motion along the width of a circle,"[62] which both indicates that the device is used by others (and almost certainly is not of his own making) and that Proclus is not one of these people since Proclus does not, and could not, refer to the motion as such.

Di Bono is certainly the most thoughtful skeptic, and his skepticism is nuanced and tempered. As an alternative to an Islamic connection (which he does not reject out of hand), he proposes that Copernicus, with the same aim of resolving the issues of irregular motion in Ptolemy's models, basically came up with the same set of devices and planetary models. Although their models are quite different from those of Copernicus, despite their use or reference to some version of Ṭūsī's couple, he proposes the same for Amico and Fracastoro. What is ironic here is that Di Bono begins his article insisting on examining the differences between the various models and their uses among the different astronomers he examines. As he puts it: "Moreover, as in this case even marginal similarities or differences may be of relevance, it is of the utmost importance not to cause such differences to disappear in the reduction to the mathematical formalism in use today."[63] But in the conclusion of the article, where his need to reduce these differences in order to argue against transmission and for multiple rediscovery (or parallel development), he falls back upon Neugebauer's point that "The mathematical logic of these methods is such that the purely historical problem of the contact or transmission, as opposed to independent discovery, becomes a rather minor one."[64] But the problem with this position is that the differences di Bono is so insistent on earlier in his article fade to irrelevance since the "internal logic" supersedes any attempt to understand the historical developments involved. And these are not insignificant, as we shall see. Yet another problem with di Bono's position is that none of his European actors has left any hint that they developed the basic devices on their own.

[61] Nicolaus Copernicus, *On the Revolutions*, trans. Edward Rosen (Baltimore: Johns Hopkins University Press, 1978), p. 279.
[62] Copernicus, *On the Revolutions,* p. 126 (in III.4, where it was crossed out in the autograph, and in III.5 where it was left in).
[63] Di Bono, "Copernicus, Amico, Fracastoro and Ṭūsī's Device," p. 133.
[64] Ibid., p. 149 (referring to an earlier reference to Neugebauer's quote in fn. 78).

And where we do have a discussion of sources, namely in *De revolutionibus*, Copernicus on the one hand makes a somewhat irrelevant gesture toward Proclus (which has all the hallmarks of a humanist need to pad his text with a classical reference), and on the other, as we have seen, refers to others who have used the device. So di Bono's contention that "the reciprocation device…could equally well have derived from an independent reflection [by Copernicus] on these same problems" seems to be undermined by what evidence is at hand.

A more recent skeptic is André Goddu, who agrees with di Bono's skepticism about an Islamic influence but is equally skeptical about di Bono's suggestion of a Paduan source. Instead he proposes Oresme as the ultimate source of the reciprocating device in Europe, someone di Bono does not mention in his own, wide-ranging article. As we have seen, Oresme does indeed describe a reciprocation device, but it is rather different than the one Goddu envisions.[65] Be that as it may, Goddu proposes the following: "The path to Copernicus would have proceeded from Oresme to Hesse, Julmann, and Sandivogius, and from them to Peurbach, Brudzewo, and Regiomontanus." But in making such a proposal, Goddu has confused, or conflated, two totally different models. Henry of Hesse (ca. 1325-1397), a certain magister Julmann (alive in 1377), Albert of Brudzewo (1445-1495), and perhaps Peurbach are not describing ("using" would be misleading here) some version or other of the Ṭūsī couple but rather something like Ibn al-Haytham's Eudoxan-couple (see above). As for Sandivogius (fl. 1430), what is being put forth is an additional epicycle for the moon that would counter the epicycle's motion that should allow us to see both faces of the moon, something that is not observed.[66] Goddu seems to be depending mainly on J.L. Mancha for his information on Hesse, Julmann, Peurbach, and Brudzewo, but Mancha makes it very clear that what they are dealing with is Ibn al-Haytham's Eudoxan-couple, not the Ṭūsī-couple. Thus when Goddu seeks to make Oresme the source for Hesse and subsequent writers, he is making a fundamental mistake, namely having something that is likely to have been some sort of Ṭūsī device be the source for a totally different type of model. For Oresme was seeking to produce rectilinear motion from circular motion, whereas most of the

[65] See above; note again that Goddu dismisses out of hand Kren's mostly correct reconstruction.
[66] Grażyna Rosińska, "Naṣīr al-Dīn al-Ṭūsī and Ibn al-Shāṭir in Cracow?" *Isis* 65, 2 (Jun., 1974): 239-243, claims that Brudzewo owes his two-sphere model for the moon to Sandivogius, but this is far from clear. Sandivogius seems to be proposing one additional orb (not two) for the moon and for an entirely different purpose, namely to keep its single face oriented toward the observer.

other authors Goddu deals with (excepting Copernicus, of course) are simply reporting a way to physicalize the small circle motion of Ptolemy's latitude theory, or using the same device for the oscillation of the lunar apogee due to the moon's prosneusis point.[67] That Goddu claims that an adaptation by Copernicus of the Eudoxan model that Brudzewo describes is equivalent to the wholesale incorporation of Ibn al-Shāṭir's models in the *Commentariolus* is, to say the least, bizarre in the extreme.[68]

Empirical Evidence for Transmission

Both di Bono and Goddu ask for more evidence for transmission before passing judgment. This is a fair comment and in what follows we present some of the evidence that has been discovered over the past 25 years or so.[69] We will divide this up into different pathways that transmission did or could have taken.

1) The Byzantine Route

As mentioned above, it is now clear that the Ṭūsī-couple first made its way into another cultural context through Byzantine intermediaries, first and foremost George Chioniades who

[67] This conclusion, as part of a longer study on Brudzewo, is also reached by Peter Barker in his "Albert of Brudzewo's Little Commentary on George Peurbach's 'Theoricae Novae Planetarum'," *Journal for the History of Astronomy* 44, no. 2 (2013): 125-148, esp. pp. 137-139. Barker seems unaware of Mancha's earlier work.

[68] "Experts have exaggerated the supposed identity between Copernicus's and al-Shatir's models and the Tusi couple. Di Bono explains the similarities plausibly as matters of notation and convention. Di Bono also shows that Copernicus's use of the models required an adaptation, and, we may add, *if he was capable of adapting geometrical solutions, then why not the solution in Albert's [i.e. Brudzewo's] treatise*? The question should be reconsidered." [p. 157]. One hardly knows where to begin. First, di Bono does not deal with Ibn al-Shāṭir's models. Secondly, the adaptation di Bono is speaking about (i.e. the 2-equal sphere model) already occurred with Ṭūsī, as we have seen. Thirdly, for Goddu to think that Copernicus could have simply adapted Brudzewo's cryptic and ultimately unrelated remarks to come up with the Ibn al-Shāṭir models in the *Commentariolus*, one must assume that Goddu has never examined those models.

[69] It should be noted that some of this evidence would have been available to di Bono and even more to Goddu whose book was published in 2010. It is unfortunate that the presumed lack of transmission that di Bono and Goddu point to does seem to be at work in the present when we consider how slowly the work of scholars working on Islamic science seems to get transmitted to their colleagues working on the Latin West. For example, Goddu, who is mainly concerned with Copernicus's relation to the Aristotelian tradition, has chosen to completely ignore the possible transmission from Islamic sources of a number of Copernican ideas related to natural philosophy, such as the motion of the Earth, the assertion of a non-Aristotelian astronomical physics, and the heliocentric transformation itself (summarized in Ragep, 2007).

traveled to Tabrīz ca. 1295 and studied with a certain Shams Bukharos, whom we can now identify as Shams al-Din al-Wabkanawī.[70] That this was done through translation from Persian into Greek raises some interesting issues of intercultural transmission. Was this a result of the fact that the language of trade between Byzantium and Iran was mainly in Persian? If so, Chioniades may have had an easier time finding someone to teach him Persian than Arabic. And indeed, most of the Islamic astronomical works that found their way into Greek seem to have been from Persian sources.[71] This may help us understand why an ostensibly out-of-date treatise, such as Ṭūsī's Persian *Muʿīniyya* and its appendix, the *Ḥall,* which contained the first versions of Ṭūsī's rectilinear couple and lunar model, were provided and taught to Chioniades rather than the mature versions found in Ṭūsī's later *Tadhkira*, which was in Arabic. But there could be other reasons. One of Chioniades's successors, George Chrysococces (fl. 1350), relates the following story, which was told to him by his teacher Manuel:

> ...in a short while he [i.e. Chioniades] was taught by the Persians, having both consorted with the King, and met with consideration from him. Then he desired to study astronomical matters, but found that they were not taught. For it was the rule with the Persians that all subjects were available to those who wished to study, except astronomy, which was for Persians only. He searched for the cause, which was that a certain ancient opinion prevailed among them, concerning the mathematical sciences, namely, that their king will be overthrown by the Romans, after consulting the practice of astronomy, whose foundation would first be taken from the Persians. He was at a loss as to how he might come to share this wonderful thing. In spite of being wearied, and having much served the Persian king, he had scarcely achieved his objective; when, by Royal command, the teachers were gathered. Soon Chioniades shone in Persia, and was thought

[70] Ragep, "New Light on Shams," pp. 243-245.

[71] David Pingree, *The Astronomical Works of Gregory Chioniades*, vol. 1: The Zīj al-ʿAlāʾī (Amsterdam: J. C. Grieben, 1985), p. 18. But there are certainly examples of Arabic works going into Greek: see Maria Mavroudi, *A Byzantine Book on Dream Interpretation: The Oneirocriticon of Achmet and its Arabic Sources* (Leiden, 2002); Alain Touwaide, "Arabic Urology in Byzantium," *Journal of Nephrology* 17 (2004), 583–9; and Alain Touwaide, "Arabic Medicine in Greek Translation: A Preliminary Report," *Journal of the International Society for the History of Islamic Medicine* 1 (2002): 45–53. And Joseph Leichter believes that Chioniades may have learned or improved his Arabic at some point; Joseph Gerard Leichter, "The Zīj as-Sanjarī of Gregory Chioniades: Text, Translation and Greek to Arabic Glossary," Thesis (Ph.D.), Brown University, 2004, pp. 11-12.

> worthy of the King's honor. Having gathered many treasures, and organized many subordinates, he again reached Trebizond, with his many books on the subject of astronomy. He translated these by his own lights, making a noteworthy effort.[72]

This of course reminds us, if we need reminding, that intercultural transmission at the time did take considerable effort and was not always a straightforward process. But it also teaches us that transmission was indeed possible. In this case, the transmission of the couple and models based on it is clear, since they occur in Chioniades's *Schemata.* What is less clear is under what circumstances the *Schemata* itself was further transmitted. And did other knowledge contained in the *Muʿīniyya* and the *Ḥall,* but not contained in the *Schemata*, also get transmitted? An example of this latter case would be Ibn al-Haytham's "Eudoxan couple" which, as we mentioned, was presented in a separate chapter in the *Ḥall* by Ṭūsī. Ibn al-Haytham's work itself is not extant, and the presentation in the *Tadhkira* is much more succinct. So a transmission of the Eudoxan-couple via Chioniades would provide an important link taking us to Henry of Hesse and beyond.

Now the *Schemata* is currently witnessed by 3 manuscripts, two in the Vatican (Gr. 211, ff. 106v-115r [text], ff. 115r-121r [diagrams], and Gr. 1058, 316r-321r) and one at the Laurenziana in Florence (28,17, ff. 169r-178r).[73] Vat. Gr. 211 and Vat. 1058 have diagrams, Laur. 28,17 does not.[74] In Vat. Gr. 211, one diagram represents the mathematical rectilinear version of the Ṭūsī couple (f. 116r) and another Ṭūsī's lunar model from the *Ḥall* (f. 117r), i.e. the one with six rather than seven orbs. Laur. 28.17 was copied in 1323 according to the colophon on f. 222v, but it is not clear when the manuscript arrived in Italy. On the other hand, Vat. Gr. 211 was copied in the early fourteenth century and was recorded in the Vatican inventory of 1475; Vat. Gr. 1058 was copied in the middle of the fifteenth century and was perhaps in the Vatican inventory of 1475 but certainly, according to Pingree, in the ca. 1510 inventory. This provides us with evidence that the work, with diagrams, was available in Italy as

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72 Raymond Mercier, "The Greek 'Persian Syntaxis' and the *Zīj-i Īlkhānī*," *Archives internationales d'histoire des sciences* 34 (1984): 35–60, on pp. 35-36; reproduced in Leichter, "Zīj as-Sanjarī," p. 3.
73 Information on the manuscripts is from Pingree, *The Astronomical Works of Gregory Chioniades*, pp. 23-28.
74 Swerdlow and Neugebauer, *Mathematical Astronomy*, p. 48, n. 9.

early as 1475; on this basis, Swerdlow and Neugebauer have favored this Italian transmission route for the Ṭūsī couple to Copernicus, who studied and traveled in Italy between 1496 and 1503 (mainly Bologna, Padua and Rome).[75] It may be significant that Copernicus spent part of the Jubilee year 1500 in Rome, perhaps to do an apprenticeship at the Papal Curia, which would have given him access to the *Schemata*.

2) The Spanish Connection

Relations between the two main branches of Christendom were fraught, and it seems likely that one of the reasons the twelfth-century translation movement brought Greek classics into Latin via Arabic translations, rather than directly from the Greek, was that it was easier to obtain Arabic versions of Greek texts in Spain rather than Greek manuscripts from Byzantium. Thus we must be cautious before assuming that Byzantine astronomy would have made its way westward before the fifteenth century. But there is another route that could have brought the new astronomy of thirteenth-century Iran to the Latin West. There is considerable historical evidence of ongoing diplomatic activity between the court of Alphonso X in Spain and the Mongol Īlkhānid rulers of Iran. The late Mercè Comes wrote an important article on the subject and noted a number of cases of similar astronomical theories and instruments appearing in both Christian Spain and Iran during the 13th c.[76] But perhaps the most striking example of a scientific theory from Īlkhānid Iran appearing in Europe is the attempted proof of Euclid's parallels' postulate, produced in the important Tabrīz scientific milieu of the 1290s, that pops up in the work of Levi Ben Gerson in southern France, probably shortly after 1328 according to Tony Levy who made this important identification.[77] This is the proof found in the Commentary on Euclid's *Elements* published at the Medici Press in Rome in 1594 and incorrectly attributed to Ṭūsī; the proof was

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[75] An excellent summary of what is known of Copernicus's life can be found in ibid., pp. 3-32.
[76] Mercè Comes, "The Possible Scientific Exchange between the Courts of Hūlāgū and Alfonso X," in *Sciences, techniques et instruments dans le monde iranien (Xe–XIXe siècle),* études réunies et présentées par N. Pourjavady et Ž. Vesel, pp. 29–50 (Tehran, 2004). Note also that Langermann ("Medieval Hebrew Texts," p. 35) alludes to the possibility of a link between Alfonso's court and Muḥyī al-Dīn al-Maghribī, who was of Andalusian origin but spent most of his career in Syria and Iran.
[77] Tony Lévy, "Gersonide, commentateur d'Euclide: traduction annotée de ses gloses sur les *Éléments,*" in *Studies on Gersonides: A Fourteenth-Century Jewish Philosopher-Scientist*, edited by G. Freudenthal, pp. 83-147 (Leiden: E. J. Brill. 1992), esp. pp. 90-91 and 100-115.

later discussed by the Italian mathematician Giovanni Saccheri.[78] Now if something as complicated as this parallels' postulate proof could travel from Iran to Avignon in 25 years or so, then presumably Ṭūsī's couple, which as we have seen had already been translated into Greek, could make it to France as well and be available for Nicole Oresme. As mentioned above, Ibn al-Haytham's Eudoxan couple is a bit more difficult to trace, but the fact that Chioniades would have no doubt encountered it in his studies of the *Ḥall* provides another plausible vehicle of transmission.

3) The Jewish Link

As we see with Gersonides, perhaps the most important agents of transmission from Islam to Christendom were Jewish scientists and mathematicians. Recent work by Tzvi Langermann and Robert Morrison has been groundbreaking in shedding light on a host of characters involved in this transmission. In addition to bringing Avner de Burgos's proof of the Ṭūsī couple to our attention, Langermann has shown that Mordecai Finzi (Italy, fifteenth century) knew the *Meyashsher ʿAqov* of Avner de Burgos, in which, as we have seen, Avner/Alfonso proved that one could produce continuous straight line oscillation by means of a Ṭūsī couple. According to Langermann, Finzi clearly knows of the *Meyashsher ʿAqov,* as indicated by his copying of the interesting conchoid construction found in Avner's text.[79] It seems reasonable to assume, as Langermann does, that Finzi knew the other parts of the *Meyashsher ʿAqov,* including the Ṭūsī couple proof. Furthermore, Finzi had extensive contacts with Christian scholars as he notes in several places in his works and translations.[80] Thus here we have a Jewish scholar who most likely knew of the Ṭūsī couple in contact with north Italian mathematicians a generation or so before Copernicus would be in the neighborhood.

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[78] Thomas Heath, tr. and comm.,*The Thirteen Books of Euclid's Elements*, 2nd ed., 3 vols. (New York: Dover Publications, 1956); see also Robert Jones, "The Medici Oriental Press (Rome 1584-1614) and the Impact of Its Arabic Publications on Northern Europe", in *The "Arabick" Interest of the Natural Philosophers in Seventeenth-Century England*, edited by Gül Russell (Leiden: Brill, 1993), pp 88-108.
[79] Langermann, "Medieval Hebrew Texts," pp. 34-35. Finzi's notebook into which he copied the construction is currently preserved at the Bodleian Library in Oxford.
[80] Y. Tzvi Langermann, "The Scientific Writings of Mordekhai Finzi," *Italia: Studi e ricerce sulla storia, la cultura e la letteratura degli ebrei d'Italia* **7** (1988): 7–44; for example, Finzi states that he made a "translation with the help of a non-Jew here in the city of Mantua" (p. 26) and in another context "I saw them in the Toledan Tables in the possession of a certain Christian" (p. 41).

In a paper in this volume, Robert Morrison discusses another avenue through which the Ṭūsī couple may have become know to Italian scholars via Jewish intermediaries.[81] In addition to summarizing recent work on Ibn Naḥmias, Morrison traces the interesting career of a certain Moses Galeano, alias Mūsā Jālīnūs. Galeano had ties to Crete and the Ottoman court of Sultan Bāyazīd II (r. 1481–1512) and also traveled to the Veneto around 1500. Most interesting is that Galeano knew of the work of Ibn al-Shāṭir whose models are so instrumental in the *Commentariolus*. Galeano also knew the writings of Ibn Naḥmias, whose models are quite similar to ones we find in Regiomontanus and Amico and that incorporated the Ṭūsī couple. Thus we have another route by which the Ṭūsī couple may well have found its way to Italy in the late 15th century.

4) Manuscripts Galore

Something often overlooked in discussions of the transmission of devices like the Ṭūsī couple (both within Islamic realms and interculturally) is that we are not dealing with a limited number of texts and manuscript witnesses. If we just confine ourselves to the major works in Islamic languages containing one or more versions of the Ṭūsī couple (there may indeed be others), we can currently identify the following texts; current known manuscript witnesses follow in parentheses:[82]

[81] Morrison, "Jews as Scientific Intermediaries in the European Renaissance".
[82] These numbers are based upon the Islamic Scientific Manuscripts Initiative (ISMI) database, which is being developed collaboratively by the Institute of Islamic Studies (McGill) and the Max Planck Institute for the History of Science (Berlin).

Author	Title	Manuscript Witnesses
Naṣīr al-Dīn al-Ṭūsī	Al-Tadhkira fī ʿilm al-hayʾa (Arabic)	72
Naṣīr al-Dīn al-Ṭūsī	Ḥall-i mushkilāt-i Muʿīniyya (Persian)	19
Naṣīr al-Dīn al-Ṭūsī	Taḥrīr al-Majisṭī (Arabic)	93
Naṣīr al-Dīn al-Ṭūsī	Taḥrīr al-Majisṭī (Persian translation)	3
Quṭb al-Dīn al-Shīrāzī	Al-Tuḥfa al-shāhiyya fī al-hayʾa (Arabic)	49
Quṭb al-Dīn al-Shīrāzī	Faʿalta fa-lā talum (supercommentary on Tadhkira; Arabic)	3
Quṭb al-Dīn al-Shīrāzī	Ikhtiyārāt-i Muẓaffar-i (Persian)	10
Quṭb al-Dīn al-Shīrāzī	Nihāyat al-idrāk fī dirāyat al-aflāk (Arabic)	37
Jalāl al-Dīn Faḍl Allāh al-ʿUbaydī	Bayān al-Tadhkira wa-tibyān al-tabṣira (Arabic)	1
al-Sayyid al-Sharīf ʿAlī ibn Muḥammad ibn ʿAli al-Ḥusaynī al-Jurjānī	Sharḥ al-Tadhkira al-Naṣīriyya	51
ʿAbd al-ʿAlī ibn Muḥammad ibn al-Ḥusayn al-Birjandī	Sharḥ al-Tadhkira	1
Fatḥ Allāh ibn Abū Yazīd ibn ʿAbd al-ʿAzīz ibn Ibrāhīm al-Shābarānī al-Shirwānī	Sharḥ al-Tadhkira	2
Ḥasan ibn Muḥammad ibn al-Ḥusayn Niẓām al-Dīn al-Aʿraj al-Nīsābūrī	Tawḍīḥ al-Tadhkira	53
Shams al-Dīn Muḥammad ibn Aḥmad al-Khafrī	Al-Takmila fī sharḥ al-Tadhkira	2
ʿUmar b. Daʾūd al-Fārisī	Takmīl al-Tadhkira	1

Thus conservatively speaking, we can currently identify some 400 manuscript witnesses that contain extensive discussions and diagrams of one or more versions of the Ṭūsī couple.[83] And many of these manuscript witnesses currently reside in Istanbul and other former Ottoman lands, including those in eastern Europe. Although most of the Islamic manuscripts currently in European libraries were collected after 1500,[84] there are, no doubt, Islamic scientific manuscripts that were available in various parts of Europe previous to that date.[85]

The last bit of empirical evidence for transmission is indirect but highly suggestive. Recently it has come to light that the critical proposition Copernicus used to transform the epicyclic models of Mercury and Venus into eccentric models, which is found in Regiomontanus's *Epitome*, was put forth earlier in the 15th century by ʿAlī Qushjī of Samarqand.[86] Although it is not known how Qushjī's treatise came to be known by Regiomontanus (which seems much more likely to me than independent rediscovery of the proposition), a likely candidate would be Cardinal Bessarion (d. 1472), the Greek prelate who almost became the Roman Pope. Bessarion traveled to Vienna in 1460, where he met both

[83] This is a conservative estimate since the number of extant manuscripts will surely increase as other libraries and private collections come to be catalogued and examined. We have also not taken into account that the Ṭūsī couple was likely mentioned, along with other astronomical subjects, in theological works, Quran commentaries, philosophical works and so forth; it should certainly not surprise us if something as interesting as a device for producing straight line motion from circular motion would be discussed in such works, as we have seen in the case of Oresme.

[84] George Saliba has done some interesting work on Islamic scientific manuscripts in Europe, but his examples are after 1500 (Saliba, 2007, pp. 154, 159). Saliba (pp. 159-162) points to an early copy of the *Tadhkira*, Vatican MS ar. 319, which was brought to Rome in 1623 as part of the Palatine collection, one of the spoils of the Thirty Years' war that was offered by Maximilian I of Bavaria to Pope Gregory XV. But it was certainly in Central Europe by the mid-sixteenth century, where it was used and perhaps annotated by Jakob Christmann (1554-1613), professor of Hebrew and Arabic at the University of Heidelberg; see Giorgio Levi Della Vida, *Ricerche Sulla Formazione Del Più Antico Fondo Dei Manoscritti Orientali Della Biblioteca Vaticana*, Studi E Testi (Città del Vaticano: Biblioteca apostolica vaticana, 1939), 329 ff., esp. p. 332. See also, Noel Swerdlow, "The Recovery of the Exact Sciences of Antiquity: Mathematics, Astronomy, Geography," in *Rome Reborn. The Vatican Library and Renaissance Culture*, edited by A. Grafton (New Haven, Conn.: Yale University Press, 1993), 125–67.

[85] For example, Biblioteca Medicea Laurenziana in Florence currently holds a copy of Quṭb al-Dīn al-Shīrāzī's *Nihāyat al-idrāk* (n. 269= Orientali 110) as well as two copies of his *al-Tuḥfa al-shāhiyya* (n. 279=Orientali 116[c] and n. 406=Orientali 215); in addition to Ṭūsī's models, the two works deal with those of Muʾayyad al-Dīn al-ʿUrḍī as well as Shīrāzī's own contributions to planetary theory. Unfortunately we do not know at present when these manuscripts first appeared in Italy.

[86] F. Jamil Ragep, "ʿAlī Qushjī and Regiomontanus: Eccentric Transformations and Copernican Revolutions." *Journal for the History of Astronomy* 36/4 (2005): 359-371; the diagrams found in the 1496 Venice printing of Regiomontanus's *Epitome* and in the manuscripts of Qushjī's treatise are quite similar.

Peurbach and Regiomontanus. That Qushjī's proposition occurs in the *Epitome* suggests that Bessarion is the intermediary, which is plausible since it should be remembered that he was originally from Trebizond and spend considerable time in Constantinople before its fall to the Ottomans in 1453. As such, he could have easily been in contact with Islamic scholars, who were in various centers in Anatolia including Bursa, the home of Qāḍīzade al-Rūmī, one of Qushjī's teachers and associates in Samarqand. Qushjī himself later came to Constantiople, in 1472, probably at the behest of Sultan Mehmed II. Now Bessarion was hardly the person to acknowledge the scientific achievements of Muslims; after all he came to Vienna as a legate of Pope Pius II to seek support for a crusade against the Turks to recapture Constantinople.[87] But his intense interest in reviving the Greek scientific heritage in Europe would have overcome any hesitancy he may have had about bringing cutting-edge Islamic scientific thought to his young acolytes.

Conclusion

The possible transmission of the Ṭūsī couple to Europe confronts us with a number of both practical and theoretical considerations. On a practical level, we need to trace the origins and development of the device and its appearance afterwards over several centuries. As we have seen, it is critical that we be clear which version of the couple we are talking about and how it is being used. We also have needed to chart the various pathways by which the couple was, or could have been, transmitted.

On a theoretical level, we need to deal with several implicit issues in what has gone before by way of conclusion. The first we can call the hermetically sealed civilization issue. Many comments on intercultural transmission have somehow assumed that after the twelfth-century translation movement from Arabic into Latin, the gates of transmission became closed, and European Christendom and Islam were sealed off from one another until the colonial period brought them back into contact, this time with the relative civilizational (but more importantly military) superiority reversed. This has had a number of historiographical consequences. Much

[87] Nancy Bisaha discusses Bessarion's attitudes and his relationship to European humanist scholars in her contribution to this volume, "European Cross-Cultural Contexts Before Copernicus".

of premodern European history, both medieval and early modern, is written from a Eurocentric point of view. Now in many cases this may be justified; like politics, much of history is local.[88] But not all. And here the insistence on an exclusively European-focused narrative can cause considerable distortions to the historical record. For example, discussing the development of trigonometry without bringing in the Indian introduction of sine and the subsequent development based on this innovation of the other trigonometric functions and identities in Islamic mathematics leaves out an essential part of the story.[89] In the case of much post-classical (i.e. post-1200 CE) Islamic science, the assumption is made that Europeans would have had little contact because of cultural and linguistic differences. But this assumption by European intellectual historians is belied by the extensive evidence of political, economic and cultural exchanges between various late Islamic regimes and European realms.[90] European travelers did go to various regions of the Islamic world before Islam, and there are certainly examples of Islamicate travelers in Europe.[91] But more to the point, it is also clear that Islamic scientific theories and objects did travel to Europe as we have seen: through contacts such as those between Spain and Īlkhānid Iran; through Jewish intermediaries; and through Byzantine scholars and émigrés.

The above-mentioned research by Langermann and Morrison, as well as by İhsan Fazlıoğlu and other historians of the Ottoman period, points to something often overlooked, namely the importance of the Ottoman courts of Mehmet II, the conqueror of Constantinople, and his son and successor Bāyazīd II for promoting scientific and philosophical study, which included providing patronage for Christian and Jewish, as well as Muslim scholars. Many of these Christian and Jewish scholars traveled readily between the Ottoman and Christian realms.[92]

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[88] A. I. Sabra emphasizes this point in his "Situating Arabic Science: Locality Versus Essence," *Isis* 87(1996): 654-670.
[89] For an elaboration, see F. Jamil Ragep, Review of "David C. Lindberg. *The Beginnings of Western Science*, 2nd edition (Chicago: University of Chicago Press, 2007)," *Isis* 100, no. 2 (2009): 383-385. A more global approach is taken by Glen Van Brummelen, *The Mathematics of the Heavens and the Earth : The Early History of Trigonometry* (Princeton: Princeton University Press, 2009).
[90] See, for example, Eric Dursteler, *Venetians in Constantinople: Nation, Identity, and Coexistence in the Early Modern Mediterranean* (Baltimore, Md.: Johns Hopkins University Press, 2006).
[91] Leo Africanus comes to mind.
[92] We have noted this in the case of Moses Galeano.

And it should not be forgotten that at the time the Ottomans were a European power with vast domains in Eastern and Central Europe, and had been such since the fourteenth century.

But there may have been more direct contact. Here one needs to confront the myth of a linguistically impoverished Europe; even scholars sympathetic to transmission such as Swerdlow and Neugebauer feel compelled to remark that "A direct transmission of the Arabic [texts containing the non-Ptolemaic models used by Copernicus] is *of course* extremely unlikely [italics added]."[93] But why of course? Some Europeans did know Arabic (how else could the 12th-c. translation movement have taken place?), and there is research showing that knowledge of Arabic was not unknown during the Renaissance.[94] At this point in our knowledge, we can only speculate that European astronomers either learned Arabic or worked with translators who did know enough to explain the non-Ptolemaic models of Ṭūsī, Ibn al-Shāṭir and others. But it seems to me equally speculative to assume they did not. For after all, Arabic is not all that esoteric—it is closely related to Hebrew which was certainly studied by numerous European Christian scholars—and there were dictionaries and grammars available. And perhaps most importantly, why would someone seek to start from scratch when it was certainly known in the 15th and 16th centuries that Islamic astronomers still had much to teach their European counterparts.[95] But more generally from a historiographical point of view, it seems odd that so many European historians of the medieval and early modern periods have written histories that make their subjects seem isolated, devoid of curiosity, and impervious to outside influences.[96]

The next theoretical point to pursue is "how much evidence is enough?". It is a commonplace in the history of science to trace intercultural transmission through the

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[93] Swerdlow and Neugebauer, *Mathematical Astronomy*, p. 48.

[94] See, for example, Karl H. Dannenfeldt, "The Renaissance Humanists and the Knowledge of Arabic", *Studies in the Renaissance* 2 (1955): 96-117 and George Saliba, "Arabic Science in Sixteenth-Century Europe: Guillaume Postel (1510-1581) and Arabic Astronomy", *Suhayl* 7 (2007): 115-164.

[95] This was even the case in the early 17th century; see Mordechai Feingold, "Decline and Fall: Arabic Science in Seventeenth-Century England," in *Tradition, Transmission, Transformation: Proceedings of Two Conferences on Premodern Science Held at the University of Oklahoma,* edited by F. Jamil Ragep and Sally P. Ragep (Leiden: E. J. Brill, 1996), pp. 441-469.

[96] Although things are changing, it is disheartening to note that in a tome of 681 double-columned pages, Robert Westman in his recent book on *The Copernican Question* (Berkeley: Univ. of California Press, 2011) devotes precisely one short, off-handed endnote to the "Maragha school" (p. 531, n. 136). Ṭūsī and the Ṭūsī couple are completely absent; Jews and Byzantines fare little better.

reappearance of numbers, objects, models, propositions, and even ideas that we can locate in an earlier source. In fact, one might consider it our most precise way to document intercultural transmission. The gold standard in our field is arguably Hipparchus's value for the mean synodic month (reported by Ptolemy), namely 29;31,50,8,20 days (sexagesimal). Once Franz Kugler demonstrated in the 1890's that this value came from what is now known as Babylonian System B, the argument for Greek knowledge and use of Babylonian astronomy (at least its parameters) became incontestable. As also that Hipparchus, despite what is reported by Ptolemy, did not make a recalculation using new observations. But why can we conclude this? The answer is obvious. Would anyone seriously contest that two identical values to the fourth sexagesimal place is coincidence? Now according to di Bono and Goddu, the appearance of Ṭūsī's couple, Urḍī's lemma, Ibn al-Shāṭir's models and so on in the work of Copernicus is not sufficient to prove transmission. But what makes this different than the case of Hipparchus's value for the mean synodic month? The case made by di Bono, and echoed by Goddu, is that somehow the "internal logic" is such that anyone confronting the problem of Ptolemy's irregular motions would come up with the same solutions.[97] But di Bono makes it clear that his criteria for accepting transmission are so high that even a "high number of coincidences between Copernican and Arab models" is insufficient, since it then "becomes very difficult to explain how such a quantity of models and information, which Copernicus would derive from Arab sources, has left no trace—apart from Ṭūsī's device—in the works of the other western astronomers of the time."[98] This is a curious argument; it is as if one were to say that one cannot prove plagiarism by an author unless one has more than one case.[99]

Let us now turn to the issue of "internal logic" and parallel development. In fact, what we have in Islam and in the Latin West represent two very different historical developments. The criticism of Ptolemy on various fronts, including observational ones, begins quite early in

[97] "In conclusion, we note that this same question of transmission may be reduced in significance, in that from a mathematical point of view—as Neugebauer has already noted—it is the internal logic of the methods used that leads the Arabs and Copernicus to such similar results…" (Di Bono, "Copernicus, Amico, Fracastoro and Ṭūsī's Device," p. 149).
[98] Di Bono, "Copernicus, Amico, Fracastoro and Ṭūsī's Device," fn. 77, pp. 153-154.
[99] This is not to say that Copernicus was a plagiarist, and as we have seen he did acknowledge in *De revolutionibus* that the Ṭūsī couple device he proved had been used previously. The point is that di Bono's argument is not a very strong one.

Islam;[100] and certainly by the time of Ibn al-Haytham (d. ca. 1040), we have sustained criticisms of the irregularities in Ptolemy's planetary models.[101] By the 13th century, we see a number of attempts to deal with these criticisms with alternative models employing devices consisting of uniformly rotating spheres, those of Ṭūsī, Muʾayyad al-Dīn al-ʿUrḍī, and Quṭb al-Dīn al-Shīrāzī being the most prominent; the proposal of alternative models continues for several centuries in Islam. It is important to emphasize that this is a sustained and traceable historical development; Ṭūsī and his successors knew of earlier criticisms and alternative models and explicitly sought to build upon their predecessors. Now this is precisely the "parallel development" that is missing in the Latin West. As we have seen, the Ṭūsī couple appears there in fits and starts; we do not find a sustained discussion of the "equant problem" before Copernicus,[102] and we certainly do not see a sustained, historically coherent development of alternative models. Here the evolution of Ṭūsī's various couples is instructive; from the initial discussion of the problem and announcement of a solution until he put forth his "final" versions, Naṣīr al-Dīn took 25 years, during which he presented various models that he would later revise. But this is precisely what is missing in the Latin case; there is no one for whom we can provide a story for the rationale and development—indeed "logic"—for one or more versions of the Ṭūsī couple. As we have seen, they just somehow appear. And it needs to be emphasized that no one after Ṭūsī claims to have independently discovered any of the versions of the couple, either in the Islamic world or in the Latin West.

In their alternative scenarios, both di Bono and Goddu have attempted to provide alternative "stories" but these are deeply flawed. Di Bono seeks to find the source for Copernicus's use of the Ṭūsī couple in the Paduan Aristotelian/Averroist critiques of Ptolemy.

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[100] F. Jamil Ragep, "Islamic Reactions to Ptolemy's Imprecisions," in *Ptolemy in Perspective*, edited by Alexander Jones, pp. 121-134 (Dordrecht; New York: Springer-Verlag, 2010).

[101] These include but certainly are not limited to the equant; see Ragep, Naṣīr al-Dīn, pp. 48-51.

[102] This is not to say that the equant as an issue was unknown in the Latin West; but perhaps with the limited exception of Henry of Hesse, one does not find the sustained criticism of Ptolemy's irregularities that is comparable to Ibn al-Haytham's *Doubts about Ptolemy*. (This is of course different than criticisms of Ptolemy based upon an Aristotelian/Averroist insistence on a homocentric cosmology.) This is surprisingly true even in the generation before Copernicus; as Dobrzycki and Kremer put it, "We know of no extant text by Peurbach or Regiomontanus in which the Ptolemaic models are criticized explicitly on the grounds that they violate uniform, circular motion" (Dobrzycki and Kremer, "Peurbach and Marāgha," note 27, p. 211).

But the problem here is that such critiques generally led to quite different homocentric modeling based on a variety of techniques that are quite distinct from those of Ṭūsī and his successors. In particular, di Bono makes no attempt to explain how Copernicus could have used the epicycle-only modeling of Ibn al-Shāṭir if he had been so influenced by astronomers and natural philosophers adamantly opposed to epicycles and eccentrics. In the case of an astronomer who did come out of that tradition and who did use one version of the Ṭūsī couple, namely Amico, we have an astronomy quite different from that of Copernicus. As for Goddu's attempt to locate Copernicus's discovery and use of the Ṭūsī couple in the Aristotelian environment of Cracow, here we have what amounts to a misunderstanding. As we have seen, Brudzewo, who Goddu wishes to make the immediate predecessor for Copernicus's use of the couple, is in fact using Ibn al-Haytham's Eudoxan couple. It is true that Brudzewo does mention it in the context of the motion of the epicyclic apogee due to the moon's prosneusis point, which, interestingly enough, is one of the examples Ṭūsī uses to explain the need for the curvilinear version of his couple.[103] But again, neither Brudzewo, nor anyone else that Goddu adduces, proposes a Ṭūsi-couple device for dealing with the problem.[104] In sum, both di Bono and Goddu depend on tenuous connections that would have us believe that their actors can move from model to model without clear agency or plausible historical context. And it is this stark contrast, between a well-developed historical context that we see in Islamic astronomy in dealing with irregular motions in Ptolemaic astronomy versus the ad-hoc episodic and de-contextualized nature of "parallel" attempts in the Latin West, that in my opinion provides us with the most compelling argument for transmission of non-Ptolemaic models such as the "Ṭūsī-couple" from Islam to Europe before the sixteenth century.[105]

Given what we know, it would seem that one possible scenario is that Copernicus was indeed influenced by Brudzewo's comments to pursue the problem of the moon's epicyclic apogee. And perhaps he realized at some point that what was needed was a curvilinear oscillation on the epicycle's circumference, as Ṭūsī had before him. Then while in Italy, he somehow encountered, through one of the routes outlined above, one or more versions of the Ṭūsī couple that he would subsequently use. But it is also clear that he was not overly interested in the

[103] Ragep, Naṣīr al-Dīn, 1: 208-213.
[104] Barker, "Albert of Brudzewo's Little Commentary," pp. 137-139 comes to a similar conclusion.
[105] This is to repeat a point made more generally in my "Copernicus and His Islamic Predecessors."

complexities of the models, which would account for his use of the apocopated two-sphere (as opposed to the full three-sphere) version in the *Commentariolus*. And by the time of composing *De revolutionibus*, he was willing to make a further simplification by using Ṭūsī's two-circle version even though it did not fulfill the need for either a full-scale physical model for rectilinear motion or a version that could produce true curvilinear oscillation.

In summary, it would seem that, as put so perceptively by Dobrzycki and Kremer, "We may be looking for a means of transmission both more fragmentary and widespread than a single treatise…".[106] And certainly by the time Copernicus wrote *De revolutionibus*, one version or another of the Ṭūsī couple would have been available in the Latin West for several centuries; in other words, it had become old hat. So perhaps Copernicus, the man from Turun, felt no need to worry about its origins, whether in Tūn or elsewhere, and could, without qualms, cross out the redundant remark in his holograph that "some people call this motion along the width of a circle."

[106] Dobrzycki and Kremer, "Peurbach and Marāgha," p. 211.

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